Al-Anbar University

College of Engineering

Mechanical engineering Department

Engineering Mechanics

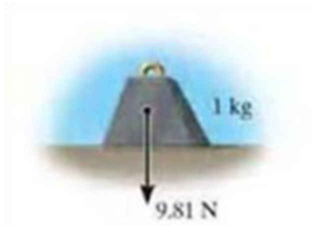
Statics

Syllabus:

- General principles
- Force vectors
- Equilibrium of a particle
- Force system resultants
- Equilibrium of a Rigid Body
- Structural Analysis
- Internal Forces
- Friction
- Center of Gravity and Centroid of Areas
- Moments of Inertia (Second Moment of Areas)

Weight: Weight refers to the gravitational attraction of the earth on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

$W = mg$ Measurements give $\boldsymbol{g} = \mathbf{9.\ 8066}\ \boldsymbol{m\ s}\ \mathbf{2}$ Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2 kg body weights 19.62 N, and so on. As shown in



Units of Measurement:

• SI units: The international System of units. Abbreviated SI is a modern version which has received worldwide recognition. As shown in Tab 1.1. The SI system defines length in meters (m), time in seconds (s), and mass in kilograms (kg). In the SI system the unit of force, the Newton is a derived unit. Thus, 1 Newton (N) is equal to a force required to give 1 kilogram of mass and acceleration of 1 $m/\ s^2$.

• US customary: In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in pounds (lb). The unit of mass, called a slug, 1 slug is equal to the amount of matter accelerated at 1 $\boldsymbol{ft}\ /\boldsymbol{s}^2$ when acted upon by a force of 1 lb (1 $slug$ = 1 $lb\ s^2$/ ft).

Name	Length	Time	Mass	Force
International Systems of Units SI	meter	seconds	kilogram	Newton
	m	s	kg	N*
US Customary FPS	foot	second	Slug*	pound
	ft	S	Ib.s^2/ft	Ib
*Derived unit				

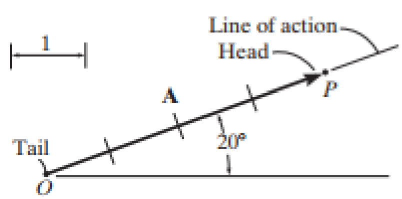
Quantities	**Unit of Measurement (FPS)**	**equals**	**Unit of Measurement (SI)**
Force	lb		4.448 N
Mass	Slug		14.59 kg
Length	ft		0.3048 m

Prefixes: When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix. Some of the prefixes used in the SI system are shown in Table 1.3. Each represents a multiple or submultiples of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place. For example, 4000000N=4000kN (kilo-newton) = 4MN (mega-newton), or 0.005m=5mm (millimeter).

	Exponential form	**Prefix**	**SI Symbol**
Multiple			
1000000000	10^9	Giga	G
1000000	10^6	Mega	M
1000	10^3	kilo	K
Submultiple			
0.001	10^{-3}	mili	m
0.000001	10^{-6}	micro	µ
0.000000001	10^{-9}	nano	n

A scalar is any positive or negative physical quantity that can be completely specified by its magnitude.

A vector is any physical quantity that requires both a magnitude and direction for its complete description. A vector is shown graphically by an arrow. The length of the arrow represents the magnitude of the vector, and a fixed axis defines the direction of its line of action .The head of the arrow indicates the sense of direction of the vector.



Procedure for Analysis

Problems that involve the addition of two forces can be solved as follows:

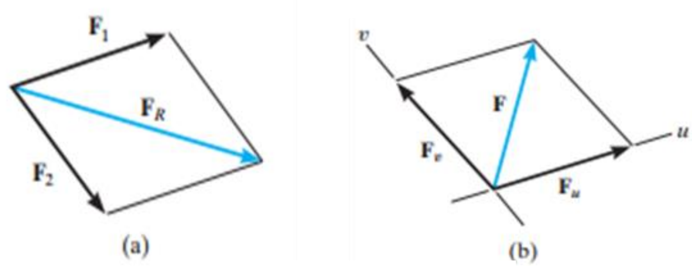
Parallelogram Law. Two "component" forces F_1 and F_2 shown in figure below add according to the parallelogram law, yielding a resultant force F_R that forms the diagonal of the parallelogram.

• If a force F is to be resolved into components along two axes u and v, then start at the head of force F and construct lines parallel to the axes, thereby forming the parallelogram. The sides of the parallelogram represent the components, F_u and F_v.

• Label all the known and unknown force magnitudes and the angles on the sketch and identify the two unknowns as the magnitude and direction of FR, or the magnitudes of its components. Trigonometry.

• Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.

• From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines.

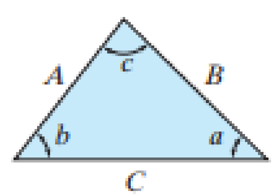


Cosine Law:

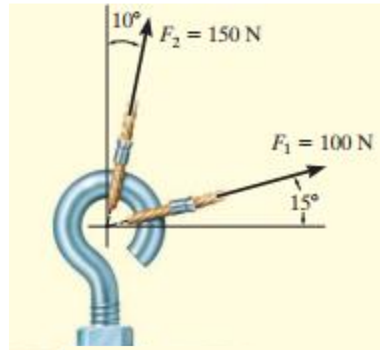
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine Law:

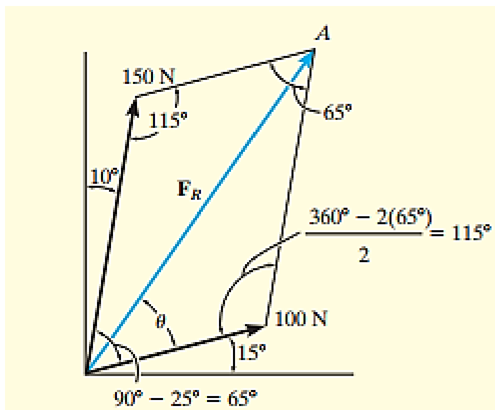
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

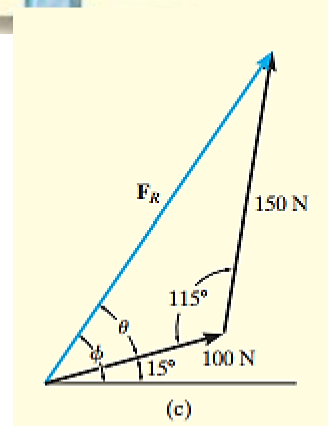


Example 1.1*:* Determine the magnitude and direction of the resultant force for the forces shown in the figure.



Solution*:* The parallelogram is formed by drawing a line from the head of $\mathbf{F_1}$ that is parallel to $\mathbf{F_2}$, and another line from the head of $\mathbf{F_2}$ that is parallel to $\mathbf{F_1}$. The resultant force $\mathbf{F_R}$ extends to where these lines intersect at point A. The two unknowns are the magnitude of $\mathbf{F_R}$ and the angle **θ** (theta).





$$F_R = \sqrt{(100)^2 + (150)^2 - 2 * 100 * 150 cos\, 115}$$

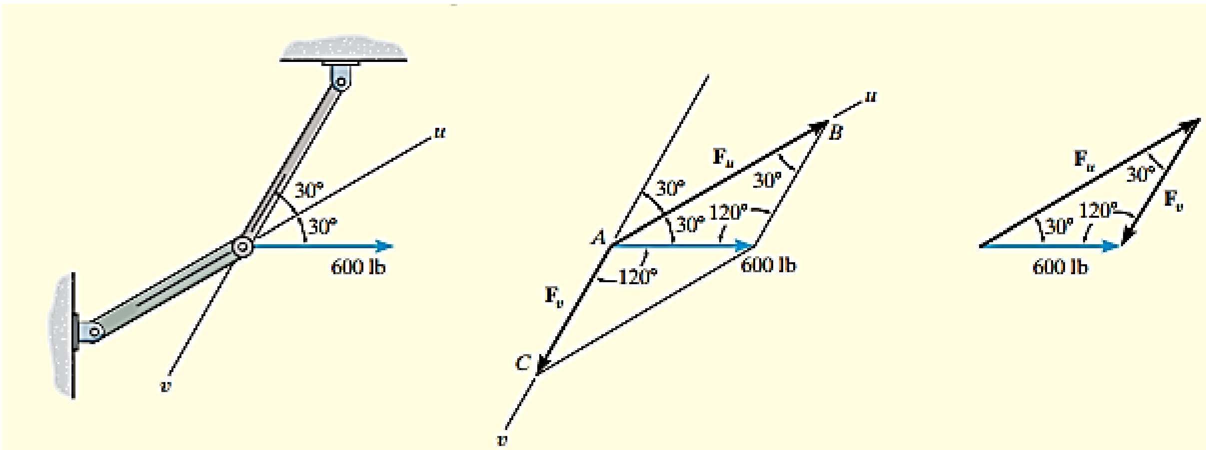
$$=212.6\ N$$

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115} \quad \rightarrow \theta = 39.8^0$$

Thus, the direction Φ (phi) of $\mathbf{F_R}$, measured from the horizontal, is

$$\Phi = 39.8^0 + 15.0^0 = 54.8^0$$

Example 1.2: Resolve the horizontal 600-lb force shown in Fig into components acting along the u and v axes and determine the magnitudes of these components.



Solution:

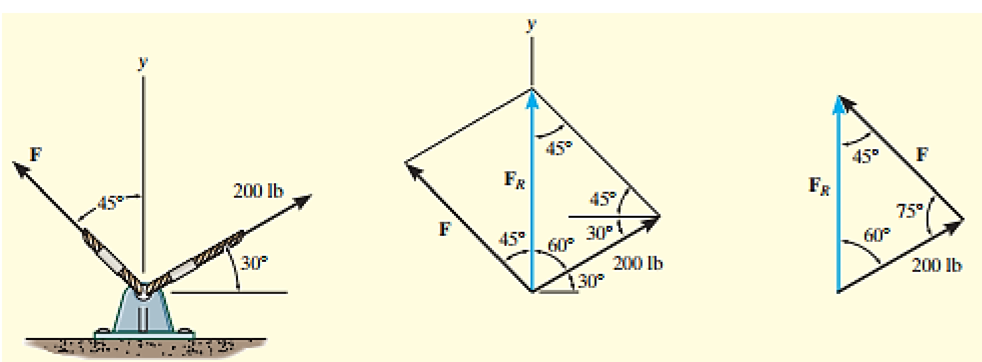
The parallelogram is constructed by extending a line from the head of the 600-lb force parallel to the v axis until it intersects the u axis at point B. The arrow from A to B represents **F_u**. Similarly, the line extended from the head of the 600-lb force drawn parallel to the u axis intersects the v axis at point C, which gives **F_v**.

$$\frac{F_u}{\sin 120} = \frac{600}{\sin 30} \quad \rightarrow F_u = 1039\ Ib$$

$$\frac{F_v}{\sin 30} = \frac{600}{\sin 30} \quad \rightarrow F_v = 600 Ib$$

Example 1.3: Determine the magnitude of the component force **F** and the magnitude of the resultant force **F_R** if **F_R** is directed along the positive **y** axis.

Solution:



$$\frac{F}{\sin 60} = \frac{200}{\sin 45} \quad \rightarrow F_u = 245\ Ib$$

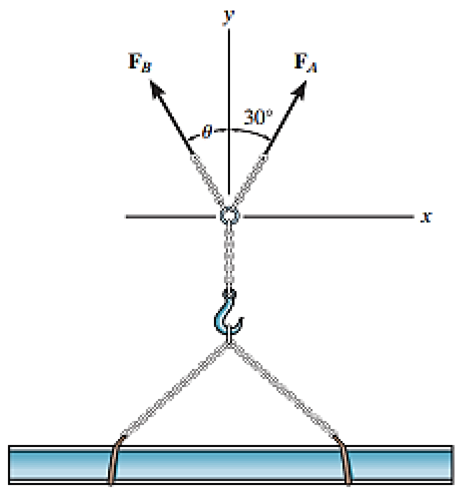
$$\frac{F_R}{\sin 75} = \frac{200}{\sin 45} \quad \rightarrow F_u = 273\ Ib$$

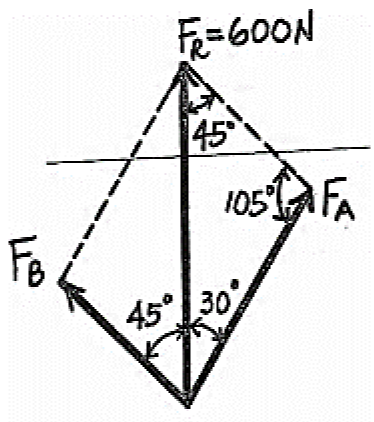
Example 1.4: The beam is to be hoisted using two chains. Determine the magnitudes of forces **F** $\mathbf{F_A}$ and $\mathbf{F_B}$ acting on each chain in order to develop a resultant force of 600 N directed along the positive **y** axis. Set **θ** = 45°.

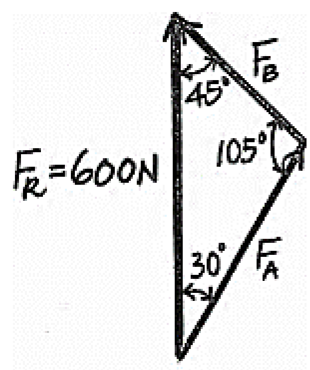
Solution:

$$\frac{F_A}{\sin 45} = \frac{600}{\sin 105} \quad \rightarrow F_A = 439\ Ib$$

$$\frac{F_B}{\sin 30} = \frac{600}{\sin 105} \quad \rightarrow F_B = 311\ Ib$$



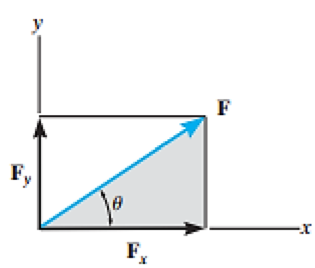


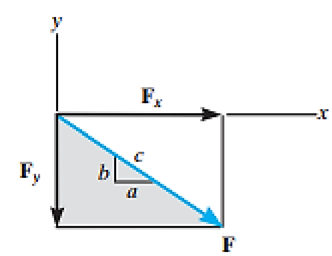


Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called rectangular components. For analytical work we can represent these components in one of two ways, using either scalar notation or Cartesian vector notation.

$\mathbf{F_x}$ = F cos θ and $\mathbf{F_y}$ = F sin θ





Instead of using the angle θ, however, the direction of **F** can also be defined using a small "slope" triangle, as in the example shown in the figure. Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives.

$$\frac{F_x}{F} = \frac{a}{c}$$

Or

$$F_x = F(\frac{a}{c})$$

$$\frac{F_y}{F} = \frac{b}{c}$$

Or for the second figure:

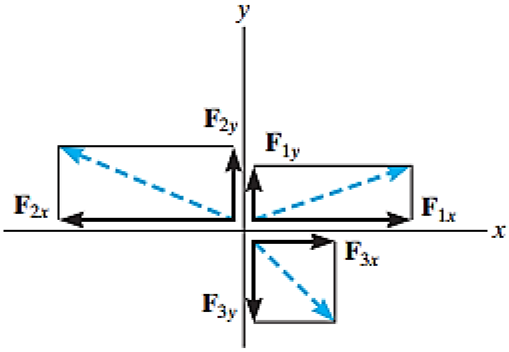
$$F_y = -F(\frac{b}{c})$$

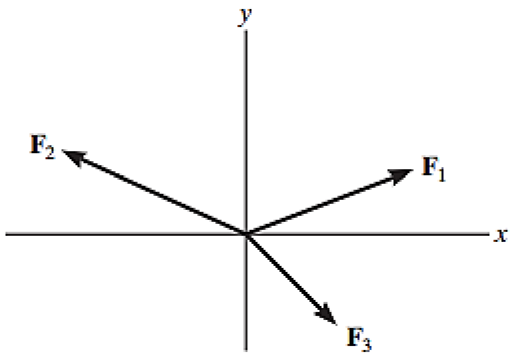
Coplanar Force Resultants:

To determine the resultant of several coplanar forces, each force is first resolved into its **x** and **y** components, and then the respective components are added using scalar algebra since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law. For example, consider the three concurrent forces in Fig. 2–17 a , which have **x** and **y** components shown in Fig.

If scalar notation is used, then from Figure, we have:

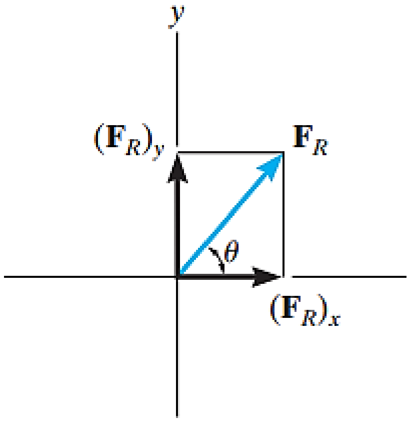
$$(\overset{+}{\rightarrow}) \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(+\uparrow) \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$





We can represent the compon1ents of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e.,

$$\boxed{\begin{aligned}(F_R)_x &= \Sigma F_x \\ (F_R)_y &= \Sigma F_y\end{aligned}}$$

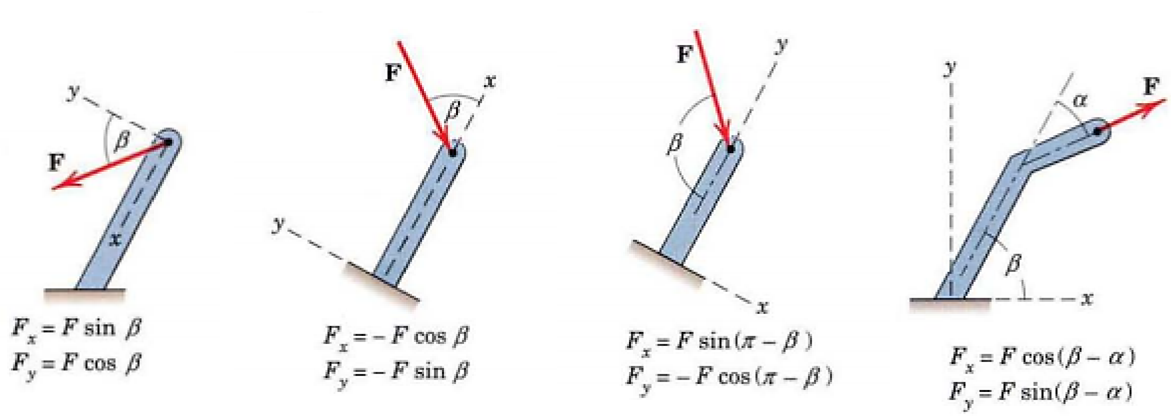


Once these components are determined, the resultant force can be determined. From this sketch, the magnitude of $\mathbf{F_R}$ is then found from the Pythagorean Theorem; that is:

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

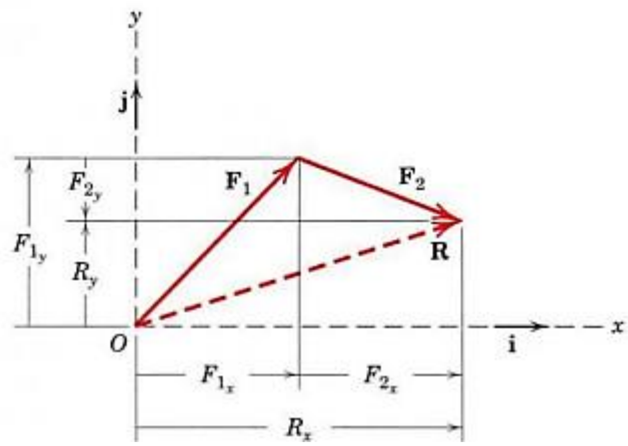
Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

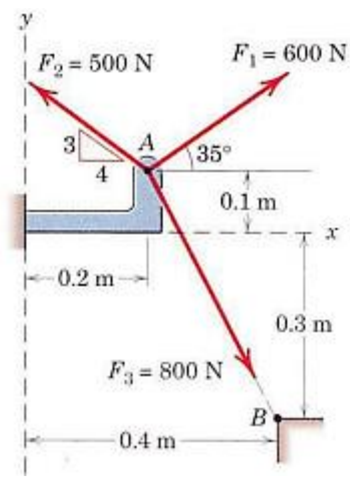


$$R_x = F_{1x} + F_{2x} = \sum F_x$$

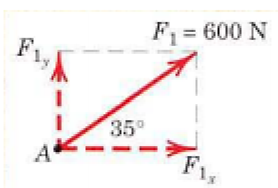
$$R_y = F_{1y} + F_{2y} = \sum F_y$$

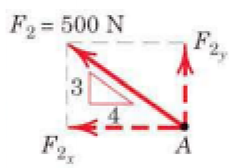


Example: The forces F_1, F_2, and F_3 all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution:





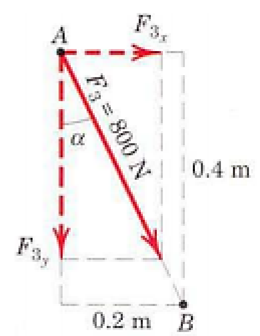
$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

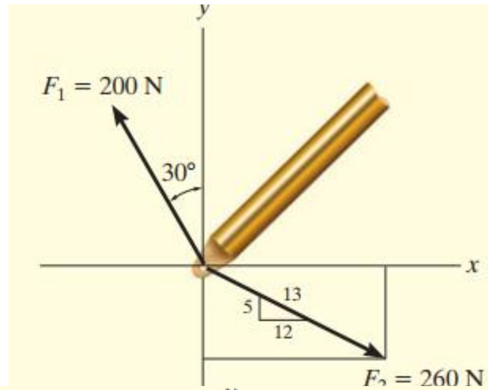
$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$



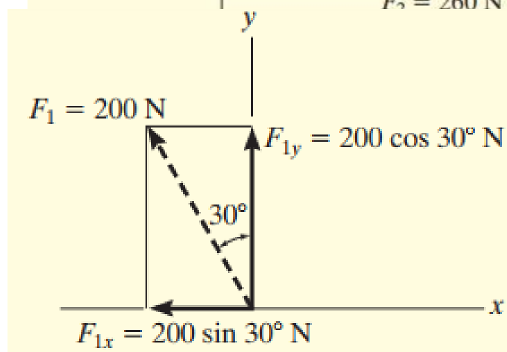
$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$
$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Then $F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6^\circ = 358$ N, $F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6^\circ = -716$ N

Example: Determine the x and y components of F1 and F2 acting on the boom shown in the Figure.



$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$
$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

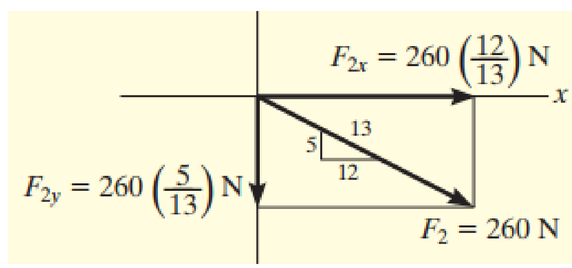


$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13}$$

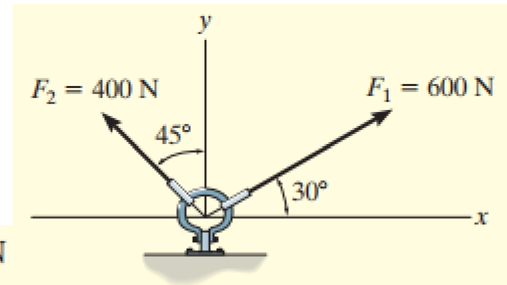
$$F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

Similarly,

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$



Example: The link as shown below is subjected to two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the magnitude and direction of the resultant force.



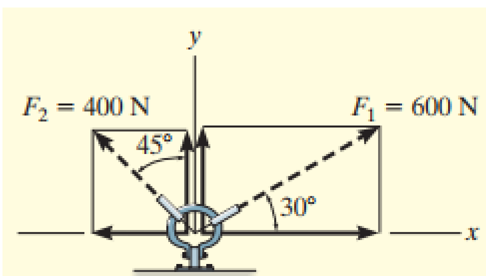
Solution:

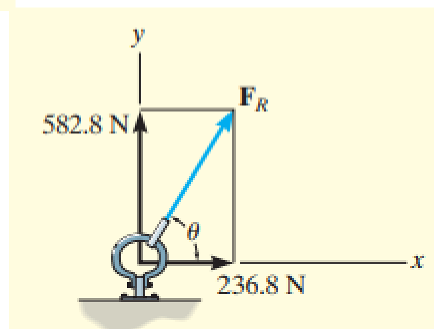
$$\xrightarrow{+}\ (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30° \text{ N} - 400 \sin 45° \text{ N} = 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30° \text{ N} + 400 \cos 45° \text{ N} = 582.8 \text{ N} \uparrow$$

The resultant force has a magnitude of:

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2} = 629 \text{ N}$$

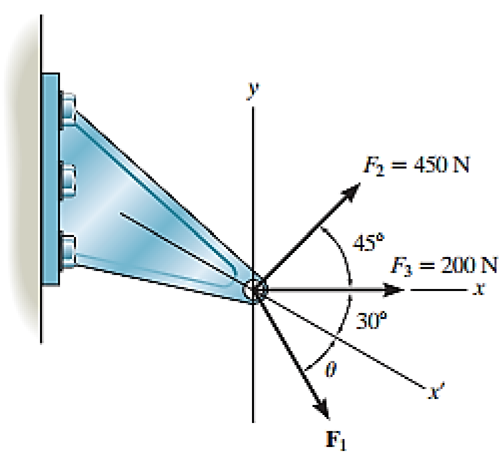




And the direction of the resultant with x axis is:

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9°$$

Example: If F_1 = 300 N and θ = 20, determine the magnitude and direction, measured counterclockwise from the x axis, of the resultant force of the three forces acting on the bracket.



Solution:

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 300 \cos 50^\circ + 200 + 450 \cos 45^\circ = 711.03 \text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -300 \sin 50^\circ + 450 \sin 45^\circ = 88.38 \text{ N}$$

$$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717 \text{ N}$$

$$\phi' \text{ (angle from } x \text{ axis)} = \tan^{-1}\left[\frac{88.38}{711.03}\right]$$

$$\phi' = 7.10^\circ$$

$$\phi \text{ (angle from } x' \text{ axis)} = 30^\circ + 7.10^\circ$$

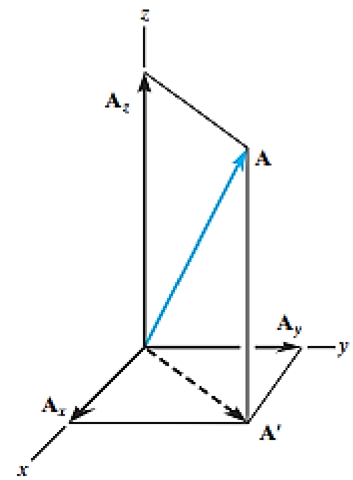
$$\phi = 37.1^\circ$$

THREE-DIMENSIONAL FORCE SYSTEMS:

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components.

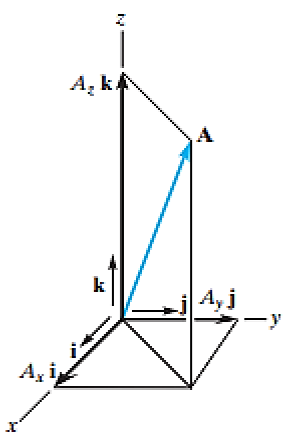
Rectangular Components of a Vector

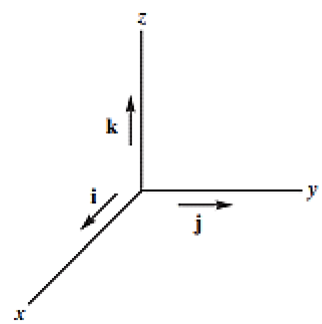
A vector A may have one, two, or three rectangular components along the x, y, z coordinate axes, depending on how the vector is oriented relative to the axes. In general, though, when A is directed within an octant of the x, y, z frame, then by two successive applications of the parallelogram law, we may resolve the vector into components as A = A´ + Az and then A´ = A_x + A_y . Combining these equations, to eliminate A´, A is represented by the vector sum of its three rectangular components,



A = A_x + A_y + A_z

- In three dimensions, the set of Cartesian unit vectors, i , j , k , is used to designate the directions of the x, y, z axes.

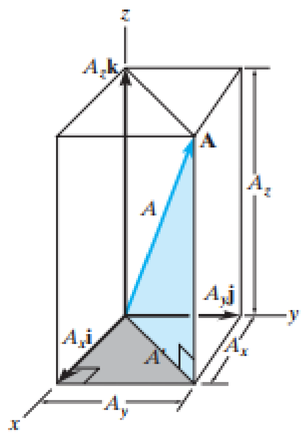




Since the three components of **A** in previous equation act in the positive i, j, and k directions, we can write **A** in Cartesian vector form as

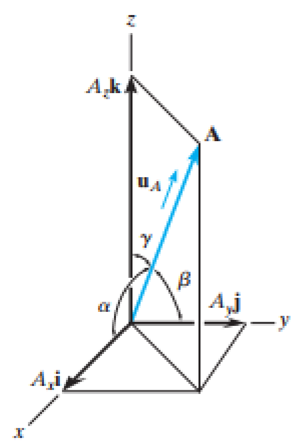
$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$

It is always possible to obtain the magnitude of A provided it is expressed in Cartesian vector form. As shown, from the blue right triangle, $A = \sqrt{A'^2 + A_z^2}$, and from the gray right triangle, $A' = \sqrt{A_x^2 + A_y^2}$ Combining these equations to eliminate A´ yields



$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Direction of a Cartesian Vector. We will define the direction of A by the coordinate direction angles **α** (alpha), **β** (beta), and **γ** (gamma), measured between the tail of **A** and the positive x, y, z axes provided they are located at the tail of A. Note that regardless of where A is directed, each of these angles will be between 0° and 180°. To determine a, b, and g, consider the projection of A onto the x, y, z axes. Referring to the blue colored right triangles shown in each figure, we have



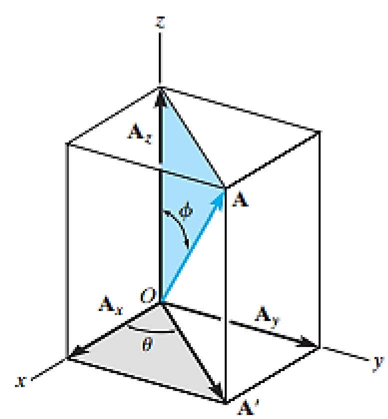
$$\boxed{\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}}$$

And

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

Sometimes, the direction of A can be specified using two angles, u and f (phi), such as shown. The components of **A** can then be determined by applying trigonometry first to the blue right triangle, which yields:

$A_z = A\cos\phi$ and $A' = A\sin\phi$



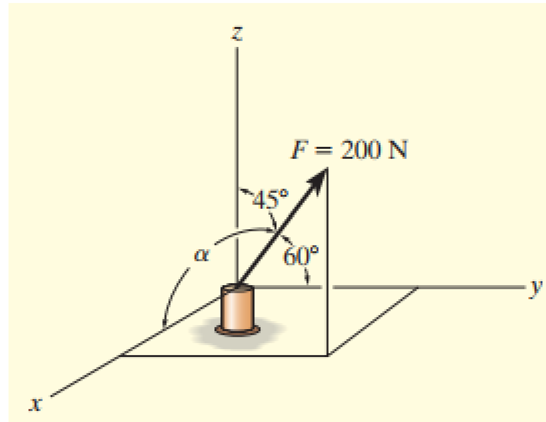
Now applying trigonometry to the gray shaded right triangle,

$$A_x = A'\cos\theta = A\sin\phi\cos\theta$$

$$A_y = A'\sin\theta = A\sin\phi\sin\theta$$

Example: determine the all the components of force F.

Solution:



The first step is to find α.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos\alpha = \sqrt{1 - (0.5)^2 - (0.707)^2} = \pm 0.5$$

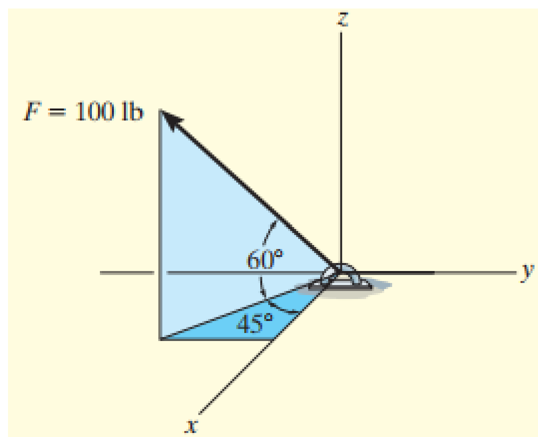
Hence, two possibilities exist, namely,

$$\alpha = \cos^{-1}(0.5) = 60^\circ \qquad \text{or} \qquad \alpha = \cos^{-1}(-0.5) = 120^\circ$$

By inspection it is necessary that α = 60, since $\mathbf{F}_x$ must be in the +x direction.

$$\begin{aligned}\mathbf{F} &= F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}\\ &= (200\cos 60^\circ\ \text{N})\mathbf{i} + (200\cos 60^\circ\ \text{N})\mathbf{j} + (200\cos 45^\circ\ \text{N})\mathbf{k}\\ &= \{100.0\mathbf{i} + 100.0\mathbf{j} + 141.4\mathbf{k}\}\ \text{N}\end{aligned}$$

Example: determine the all the components of force F.



Solution:

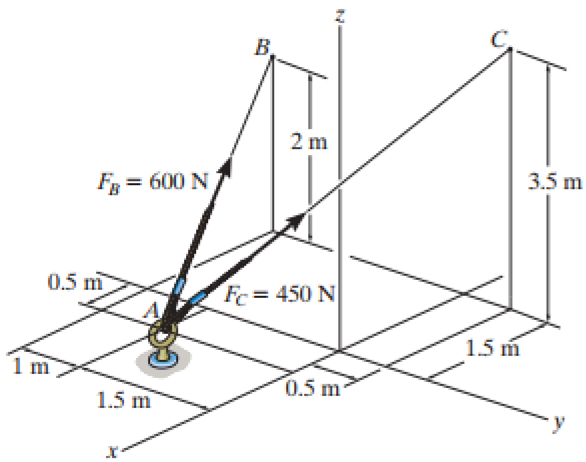
F_z = 100 sin 60 lb = 86.6 lb

F´ = 100 cos 60 lb = 50 lb

Fx = F´ cos 45 = 50 cos 45 lb = 35.4 lb

Fy = F´ sin 45 = 50 sin 45 lb = 35.4 lb

Example: find the components of forces F_B and F_C.



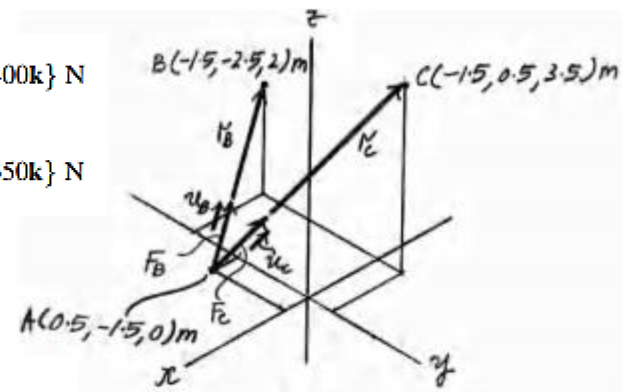
Solution:

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5-0.5)\mathbf{i} + [-2.5-(-1.5)]\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [-2.5-(-1.5)]^2 + (2-0)^2}}$$
$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

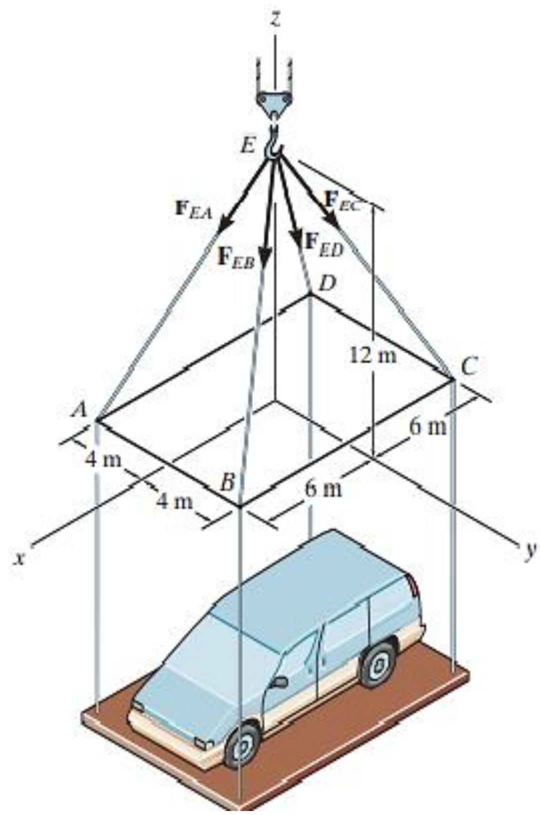
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5-0.5)\mathbf{i} + [0.5-(-1.5)]\mathbf{j} + (3.5-0)\mathbf{k}}{\sqrt{(-1.5-0.5)^2 + [0.5-(-1.5)]^2 + (3.5-0)^2}}$$
$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\ \text{N}$$



Example: Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force



Solution:

$$\mathbf{F}_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

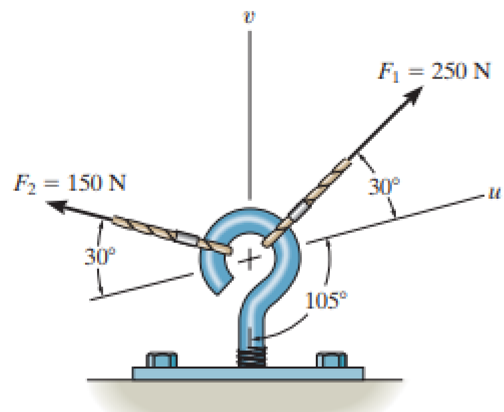
$$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\ \text{kN}$$

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

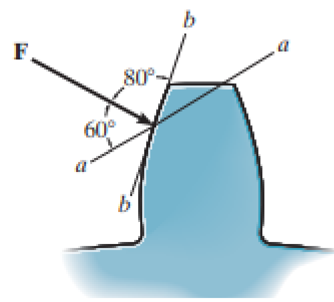
$$= \{-96\mathbf{k}\}\ \text{kN}$$

H.w:

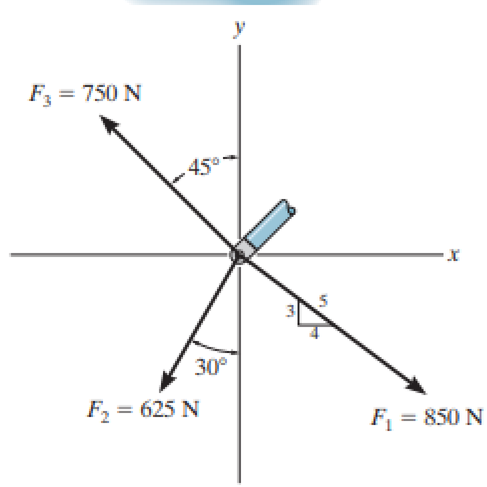
1. Resolve $\mathbf{F_2}$ into components along the u and v axes and determine the magnitudes of these components.



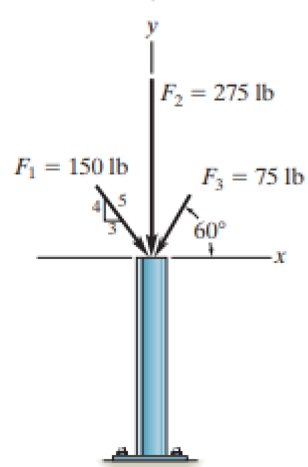
2. The component of force F acting along line aa is required to be 30 lb. Determine the magnitude of F and its component along line bb.



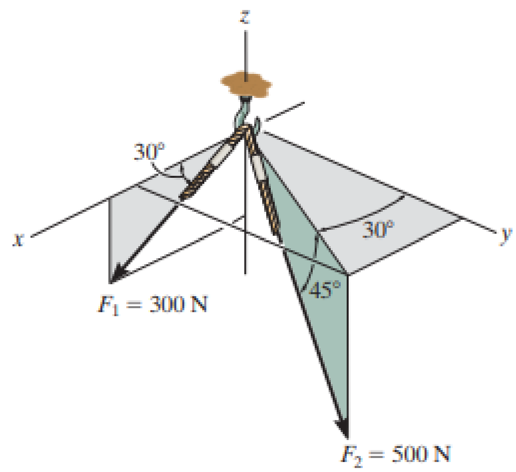
3. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



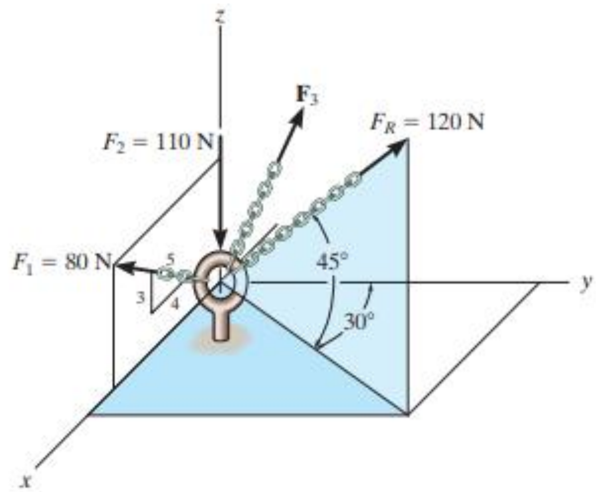
4. Express each of the three forces acting on the column in Cartesian vector form and compute the magnitude of the resultant force.



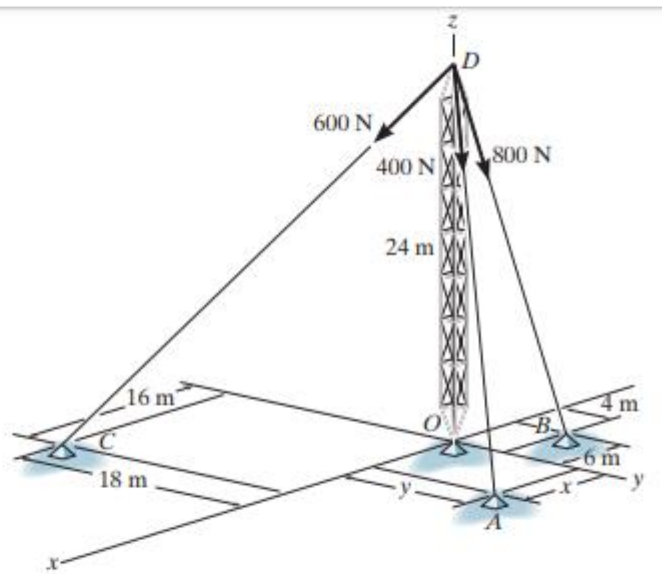
1. Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



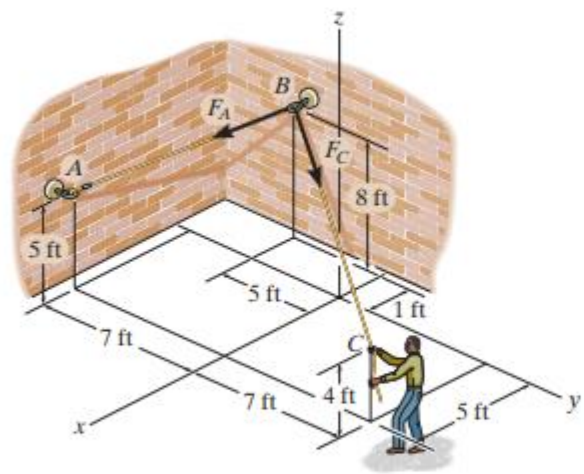
2. Three forces act on the ring. If the resultant force F_R has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force F_3. Determine the coordinate direction angles of F_1 and F_R.



3. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles a, b, g of the resultant force. Take x = 20 m, y = 15 m.



4. The man pulls on the rope at C with a force of 70 lb which causes the forces FA and FC at B to have this same magnitude. Determine the magnitude and coordinate direction angles of the resultant force acting at B.



Equilibrium of a Particle:

CHAPTER OBJECTIVES:

- To introduce the concept of the free-body diagram for a particle.
- To show how to solve particle equilibrium problems using the equations of equilibrium.

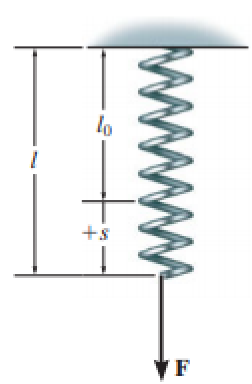
A particle is said to be in *equilibrium* if it remains at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe an object at rest. To maintain equilibrium, it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as

$$\sum F = 0$$

Where F is the vector sum of all the forces acting on the particle. Not only this equation is necessary condition for equilibrium, it is also a sufficient condition. This follows from Newton's second law of motion, which can be written as $\sum F = ma$. Since the force system satisfies the previous equation, then ma = 0, and therefore the particle's acceleration a = 0. Consequently, the particle indeed moves with constant velocity or remains at rest.

The Free-Body Diagram

To apply the equation of equilibrium, we must account for all the known and unknown forces (F) which act on the particle. The best way to do this is to think of the particle as isolated and "free" from its surroundings. A drawing that shows the particle with all the forces that act on it is called a free-body diagram (FBD).

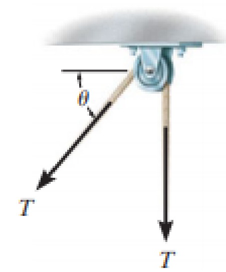


Springs. If a linearly elastic spring (or cord) of unreformed length l_0 is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it. A characteristic that defines the "elasticity" of a spring is the spring constant or stiffness **k**. The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_0$, measured from its unloaded position, is

$$\mathbf{F = k.s}$$

If s is positive, causing an elongation, then F must pull on the spring; whereas if s is negative, causing a shortening, then F must push on it. For example, if the spring as previous figure has an unstretched length of 0.8 m and a stiffness k = 500 N>m and it is stretched to a length of 1 m, so that s = l - l0 = 1 m - 0.8 m = 0.2 m, then a force F = ks = 500 N/m (0.2 m) = 100 N is needed.

Cables and Pulleys. Unless otherwise stated throughout this book, all cables (or cords) will be assumed to have negligible weight and they cannot stretch. Also, a cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable, it will be shown that the tension force developed in a continuous cable which passes over a frictionless pulley must have a constant magnitude to keep the cable in equilibrium. Hence, for any angle u, shown in figure, the cable is subjected to a constant tension **T** throughout its length.



Cable is in tension

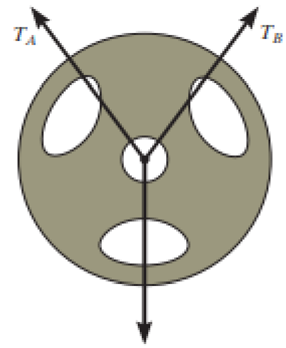
Procedure for Drawing a Free-Body Diagram:

Since we must account for all the forces acting on the particle when applying the equations of equilibrium, the importance of first drawing a free-body diagram cannot be overemphasized. To construct a free-body diagram, the following three steps are necessary.

- **Draw Outlined Shape**. Imagine the particle to be isolated or cut "free" from its surroundings by drawing its outlined shape.
- **Show All Forces**. Indicate on this sketch all the forces that act on the particle. These forces can be active forces, which tend to set the particle in motion, or they can be reactive forces which are the result of the constraints or supports that tend

to prevent motion. To account for all these forces, it may be helpful to trace around the particle's boundary, carefully noting each force acting on it.

- **Identify Each Force**. The forces that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.



Procedure for Analysis

Coplanar force equilibrium problems for a particle can be solved using the following procedure.

Free-Body Diagram.

• Establish the x, y axes in any suitable orientation.

• Label all the known and unknown force magnitudes and directions on the diagram.

• The sense of a force having an unknown magnitude can be assumed.

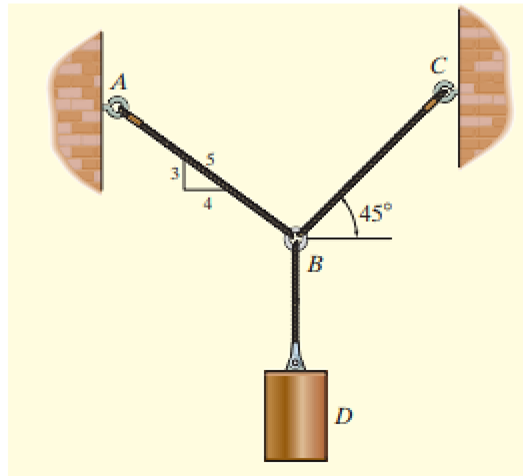
Equations of Equilibrium. • Apply the equations of equilibrium, Fx = 0 and Fy = 0.

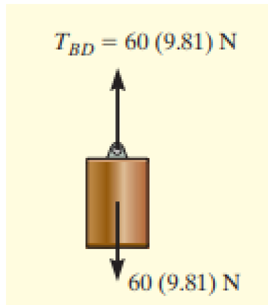
• Components are positive if they are directed along a positive axis, and negative if they are directed along a negative axis.

• If more than two unknowns exist and the problem involves a spring, apply F = ks to relate the spring force to the deformation s of the spring.

• Since the magnitude of a force is always a positive quantity, then if the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

Example: Determine the tension in cables BA and BC necessary to support the 60-kg cylinder.





SOLUTION: Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be T_{BD} = 60(9.81) N, The forces in cables BA and BC can be determined by investigating the equilibrium of ring B. The magnitudes of T_A and T_C are unknown, but their directions are known. Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

so that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

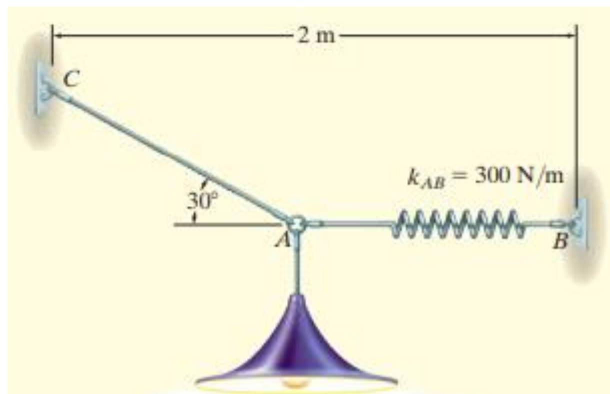
NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.

Example: Determine the required length of cord AC shown in Figure, so that the 8-kg lamp can be suspended in the position shown. The undeformed length of spring AB is l_{AB} = 0.4 m, and the spring has a stiffness of k_{AB} = 300 N/m.

Solution:

If the force in spring AB is known, the stretch of the spring can be found using F = ks. From the problem geometry, it is then possible to calculate the required length of AC . Free-Body Diagram. The lamp has a weight W = 8(9.81) = 78.5 N and so the free-body diagram of the ring at A is shown.

Equations of Equilibrium. Using the x, y axes,



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{AB} - T_{AC}\cos 30^\circ = 0$$
$$+\uparrow \Sigma F_y = 0; \qquad T_{AC}\sin 30^\circ - 78.5\text{ N} = 0$$

Solving, we obtain

$$T_{AC} = 157.0\text{ N}$$
$$T_{AB} = 135.9\text{ N}$$

The stretch of spring AB is therefore

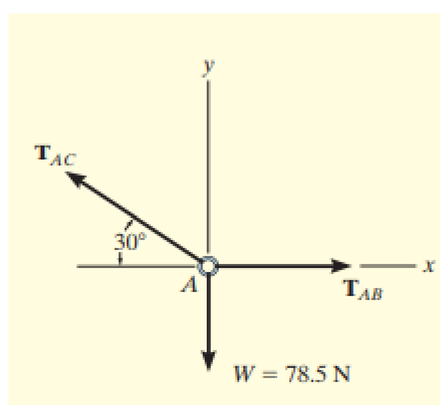
$$T_{AB} = k_{AB}s_{AB}; \qquad 135.9\text{ N} = 300\text{ N/m}(s_{AB})$$
$$s_{AB} = 0.453\text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$
$$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$$

The horizontal distance from C to B, requires

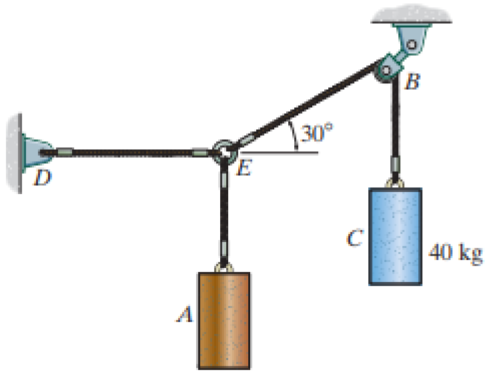
$$2\text{ m} = l_{AC}\cos 30^\circ + 0.853\text{ m}$$
$$l_{AC} = 1.32\text{ m}$$



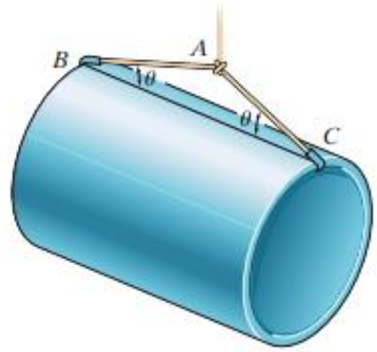
If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

H.W:

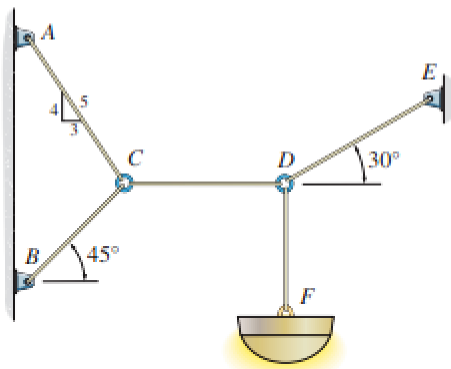
- If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.



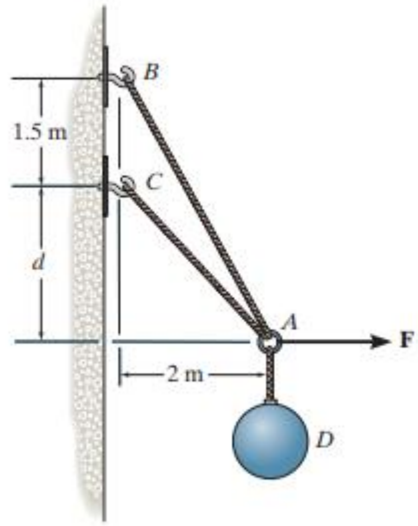
Cords AB and AC can each sustain a maximum tension of 800 lb. If the drum has a weight of 900 lb, determine the smallest angle u at which they can be attached to the drum.



- Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.



- The ball D has a mass of 20 kg. If a force of F = 100 N is applied horizontally to the ring at A, determine the dimension d so that the force in cable AC is zero.



Three-Dimensional Force Systems

In the case of a three-dimensional force system, as shown, we can resolve the forces into their respective i , j , k components, so that $F_x i + F_y j + F_z k = 0$. To satisfy this equation we require

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

Procedure for Analysis

Three-dimensional force equilibrium problems for a particle can be solved using the following procedure.

Free-Body Diagram.

• Establish the x, y, z axes in any suitable orientation.

• Label all the known and unknown force magnitudes and directions on the diagram.

• The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium.

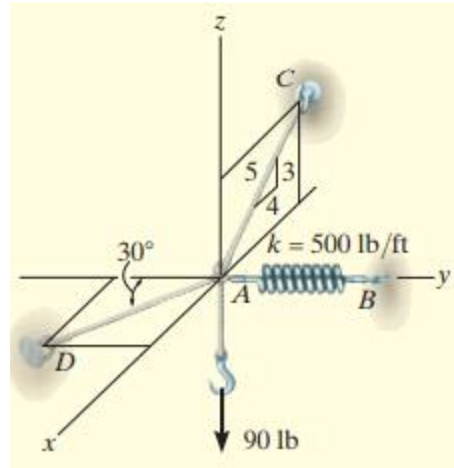
• Use the scalar equations of equilibrium, Fx = 0, Fy = 0, Fz = 0, in cases where it is easy to resolve each force into its x, y, z components.

• If the three-dimensional geometry appears difficult, then first express each force on the free-body diagram as a Cartesian vector, substitute these vectors into F = 0, and then set the i , j , k components equal to zero.

• If the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

Example: A 90-lb load is suspended from the hook shown. If the load is supported by two cables and a spring having a stiffness k = 500 lb.ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram as shown.



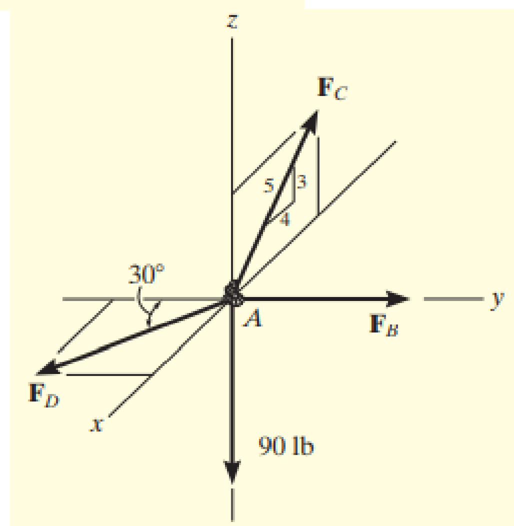
Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be used. Considering components directed along each positive axis as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30^\circ - \left(\tfrac{4}{5}\right)F_C = 0$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30^\circ + F_B = 0$$

$$\Sigma F_z = 0; \qquad \left(\tfrac{3}{5}\right)F_C - 90 \text{ lb} = 0$$

$$F_C = 150 \text{ lb} \qquad F_D = 240 \text{ lb} \qquad F_B = 207.8 \text{ lb} = 208 \text{ lb}$$



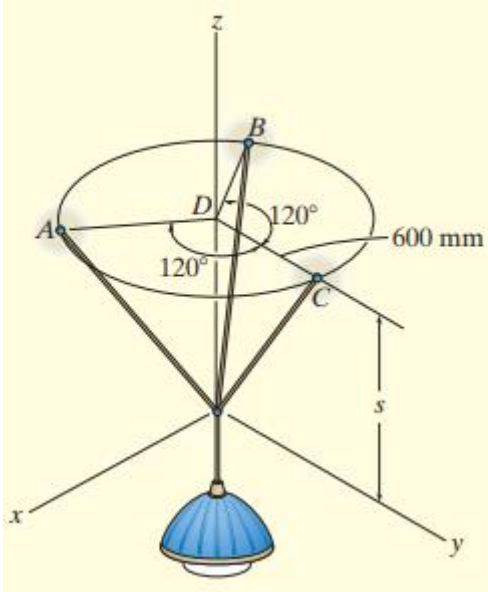
The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

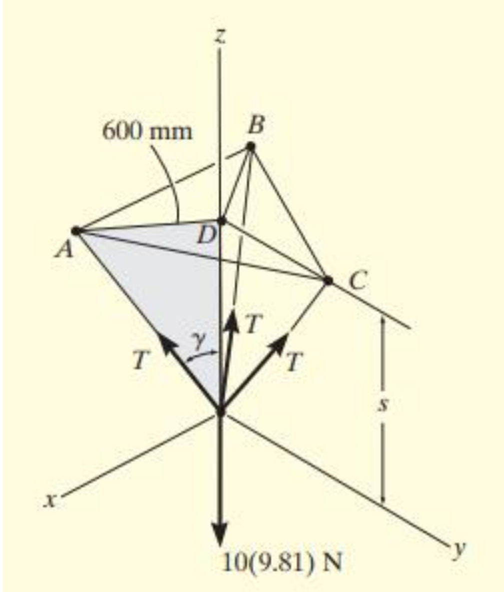
$$s_{AB} = 0.416 \text{ ft}$$

Example: The 10-kg lamp as shown is suspended from the three equal-length cords. Determine its smallest vertical distance s from the ceiling if the force developed in any cord is not allowed to exceed 50 N.



Solution:

Free-Body Diagram. Due to symmetry, the distance DA = DB = DC = 600 mm. It follows that from$\sum F_x = 0$ $\sum F_y = 0$, the tension T in each cord will be the same. Also, the angle between each cord and the z axis is γ.



Equation of Equilibrium. Applying the equilibrium equation along the z axis, with T = 50 N , we have

$$\sum F_z = 0; \qquad 3[(50\text{ N})\cos\gamma] - 10(9.81)\text{ N} = 0$$

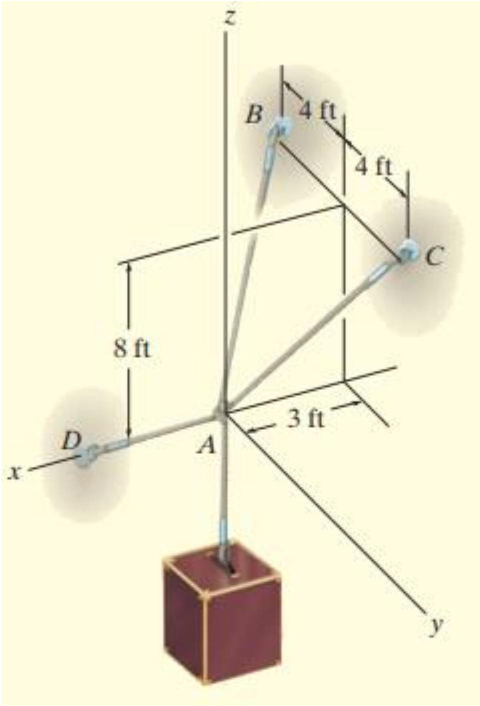
$$\gamma = \cos^{-1}\frac{98.1}{150} = 49.16^\circ$$

From the shaded triangle shown

$$\tan 49.16^\circ = \frac{600\text{ mm}}{s}$$

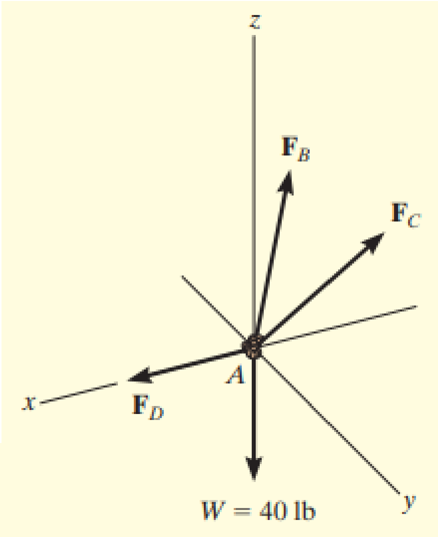
$$s = 519\text{ mm}$$

Example: Determine the force in each cable used to support the 40-lb crate as shown.



Solution:

Free-Body Diagram. As shown, the free-body diagram of point **A** is considered in order to "expose" the three unknown forces in the cables.



Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points B and C are B (-3 ft, -4 ft, 8 ft) and C (-3 ft, 4 ft, 8 ft), we have

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$
$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$
$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{i} \qquad \mathbf{W} = \{-40\mathbf{k}\}\text{ lb}$$

Equilibrium requires: $\Sigma\mathbf{F} = \mathbf{0};$

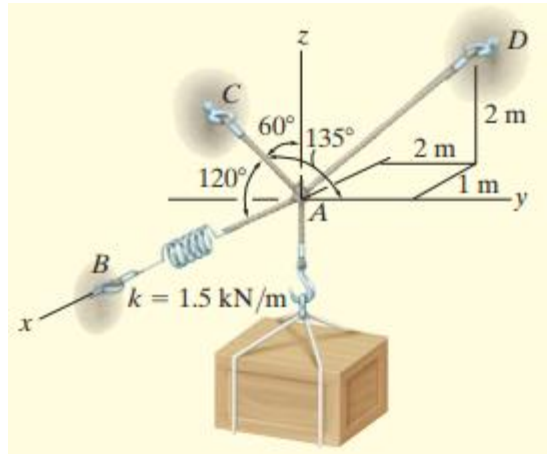
$$\mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$-0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

Equating the respective i, j , k components to zero yields:

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0$$
$$\Sigma F_y = 0; \qquad -0.424F_B + 0.424F_C = 0$$
$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0$$

$$F_B = F_C = 23.6\text{ lb}$$
$$F_D = 15.0\text{ lb}$$

Example: Determine the tension in each cord used to support the 100-kg crate as shown in Figure.



Solution:

Free-Body Diagram. The force in each of the cords can be determined by investigating the equilibrium of point A. The free-body diagram is shown in second figure. The weight of the crate is W = 100(9.81) = 981 N.

Equations of Equilibrium. Each force on the free-body diagram is first expressed in Cartesian vector form. Point D(–1 m, 2 m, 2 m) for FD, we have

$$\mathbf{F}_B = F_B\mathbf{i}$$

$$\mathbf{F}_C = F_C \cos 120°\mathbf{i} + F_C \cos 135°\mathbf{j} + F_C \cos 60°\mathbf{k}$$
$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$
$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\}\ \text{N}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$
$$-0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

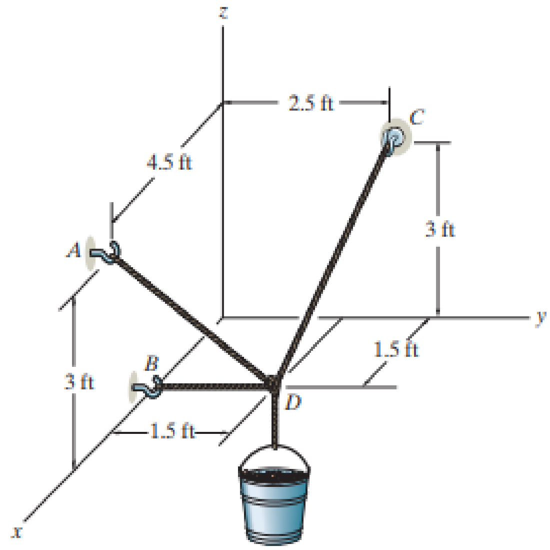
Equating the respective **i**, **j**, **k** components to zero,

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0$$
$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0$$
$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0$$

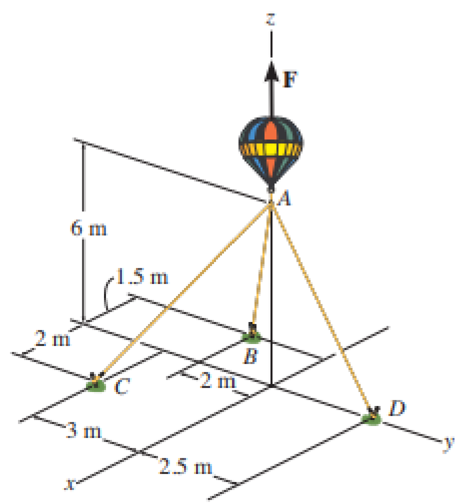
$$F_C = 813\ \text{N}$$
$$F_D = 862\ \text{N}$$
$$F_B = 694\ \text{N}$$

H.W:

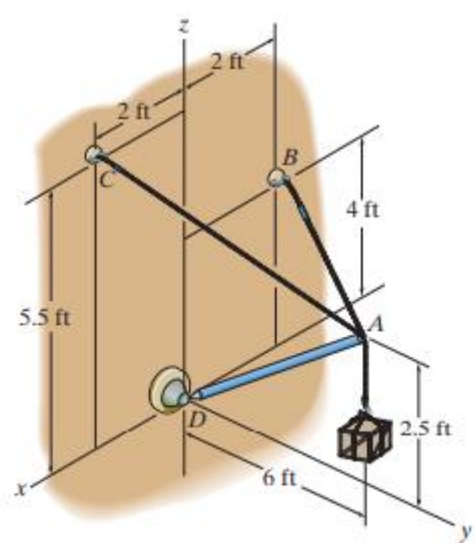
1. If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB, and DC.



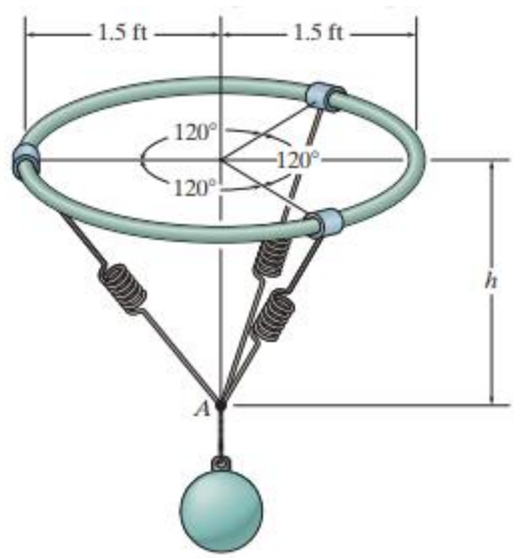
2. If each one of the ropes will break when it is subjected to a tensile force of 450 N, determine the maximum uplift force F the balloon can have before one of the ropes breaks.



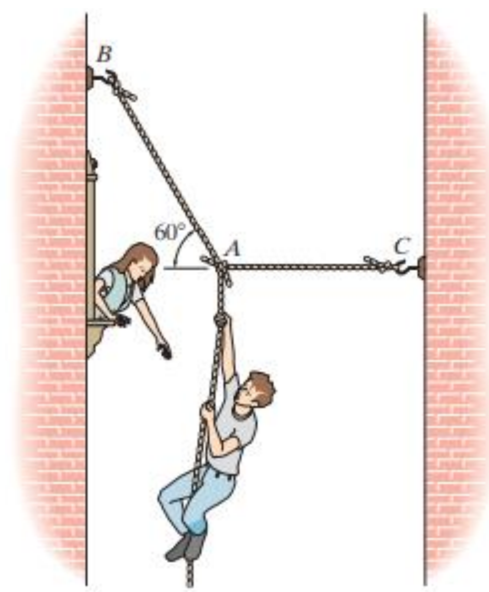
3. If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut AD?



4. The 80-lb ball is suspended from the horizontal ring using three springs each having an unstretched length of 1.5 ft and stiffness of 50 lb/ft. Determine the vertical distance h from the ring to point A for equilibrium.



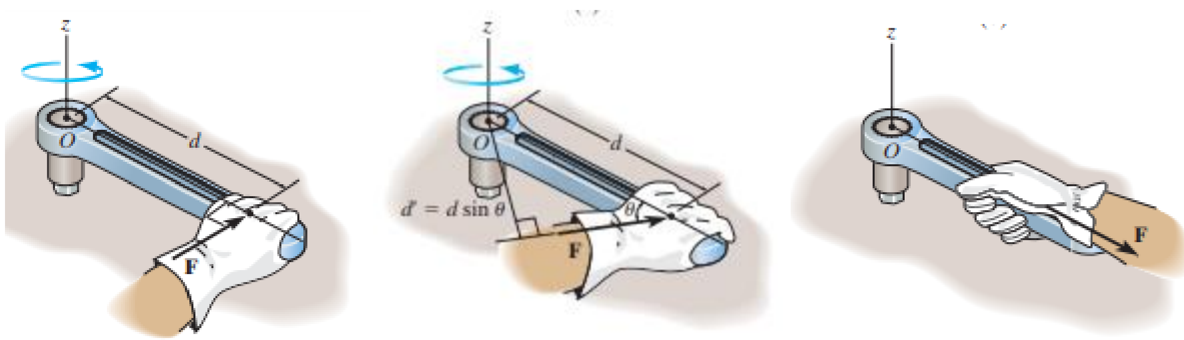
5. Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of 2 kN before it breaks. Determine if Romeo, who has a mass of 65 kg, can climb the rope, and if so, can he along with Juliet, who has a mass of 60 kg, climb down with constant velocity?



Force System Resultants:

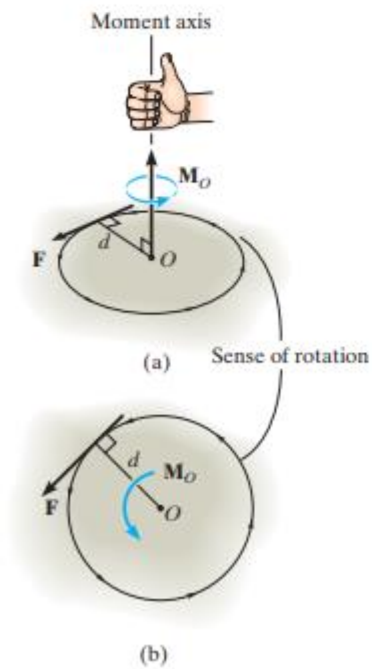
CHAPTER OBJECTIVES

- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions.
- To provide a method for finding the moment of a force about a specified axis.
- To define the moment of a couple.
- To present methods for determining the resultants of non-concurrent force systems.
- To indicate how to reduce a simple distributed loading to a resultant force having a specified location.



When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a torque, but most often it is called the moment of a force or simply the moment. For example, consider a wrench used to unscrew the bolt as shown. If a force is applied to the handle of the wrench it will tend to turn the bolt about point O (or the z axis). The magnitude of the moment is directly proportional to the magnitude of F and the perpendicular distance or moment arm d. The larger the force or the longer the moment arm, the greater the moment or turning effect. Note that if the force F is applied at an angle $\theta \neq 90$, then it will be more difficult to turn the bolt since the moment arm $d = d \sin \theta$ will be smaller than d. If F is applied along the wrench, its moment arm will be zero since the line of action of F will intersect point O (the z axis). As a result, the moment of F about O is also zero and no turning can occur.

We can generalize the above discussion and consider the force F and point O which lie in the shaded plane as shown. The moment MO about point O, or about an axis passing through O and perpendicular to the plane, is a vector quantity since it has a specified magnitude and direction.



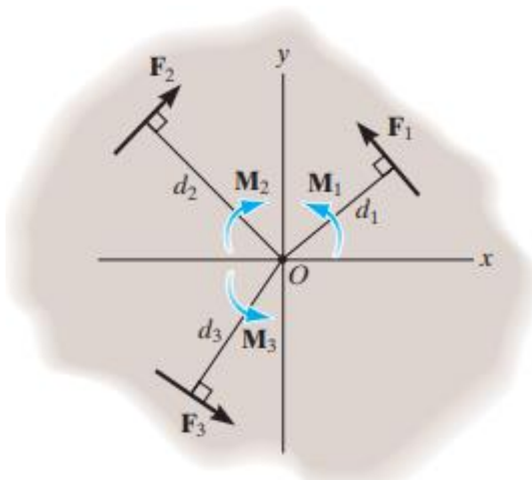
Magnitude. The magnitude of M_O is:

$$M_o = F.d$$

where d is the moment arm or perpendicular distance from the axis at point O to the line of action of the force. Units of moment magnitude consist of force times distance, e.g., N m or lb .ft.

Direction. The direction of M_O is defined by its moment axis, which is perpendicular to the plane that contains the force F and its moment arm **d**. The right-hand rule is used to establish the sense of direction of M_O. According to this rule, the natural curl of the fingers of the right hand, as they are drawn towards the palm, represent the rotation, or if no movement is possible, there is a tendency for rotation caused by the moment. As this action is performed, the thumb of the right hand will give the directional sense of M_O. Notice that the moment vector is represented three-dimensionally by a curl around an arrow. In two dimensions this vector is represented only by the curl as in Figure **b**. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

Resultant Moment.



For two-dimensional problems, where all the forces lie within the x–y plane, the resultant moment $(M_R)_O$ about point O (the z axis) can be determined by finding the algebraic sum of the moments caused by all the forces in the system. As a convention, we will generally consider positive moments as counterclockwise since they are directed along the positive z axis (out of the page). Clockwise moments will be negative. Doing this, the directional sense of each moment can be represented by a plus or minus sign. Using this sign convention, the resultant moment in Figure is therefore

$$\circlearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$

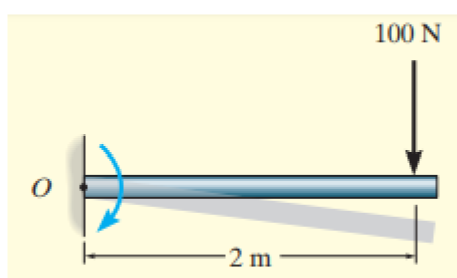
If the numerical result of this sum is a positive scalar, $(M_R)_O$ will be a counterclockwise moment (out of the page); and if the result is negative, $(M_R)_O$ will be a clockwise moment (into the page).

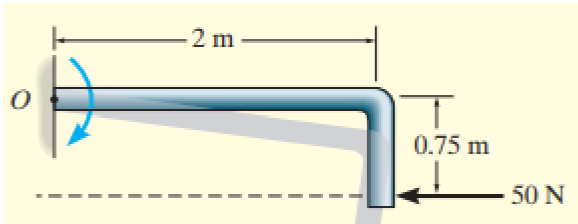
Example: For each case illustrated in Figure below, determine the moment of the force about point O.

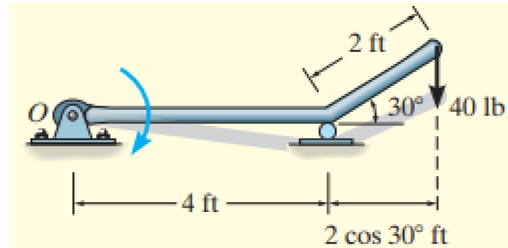
Solution:

SOLUTION: The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force? Furthermore, the orbit of the force about O is shown as a colored curl. Thus

$$M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$$

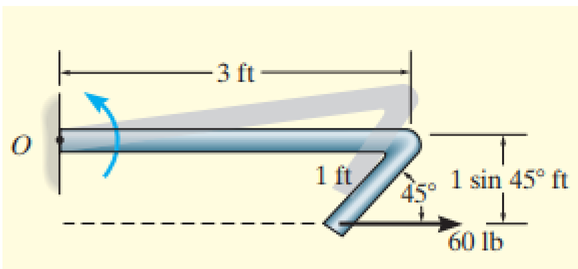




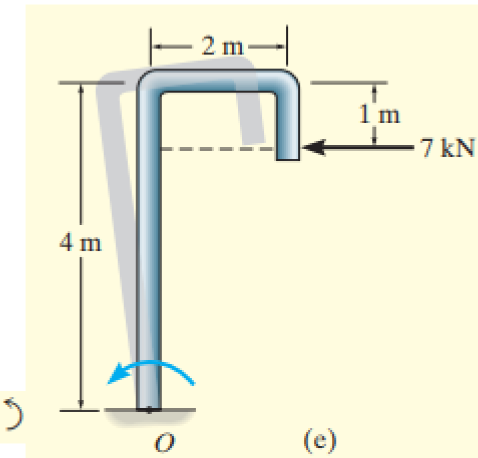


$M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m} \circlearrowright$

$M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft} \circlearrowright$

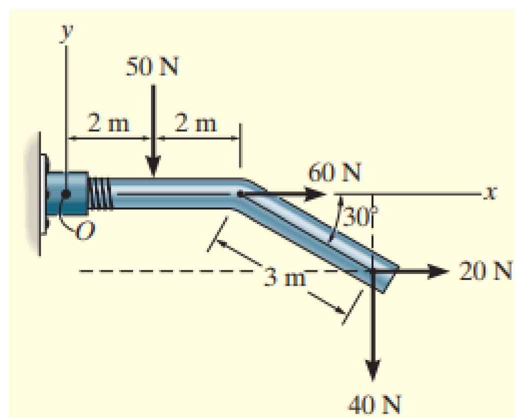


$M_O = (60\text{ lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{ lb}\cdot\text{ft} \circlearrowleft$



$M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m} \circlearrowleft$

Example: Determine the resultant moment of the four forces acting on the rod shown in Figure about point O.



Solution:

Assuming that positive moments act in the +k direction, i.e., counterclockwise, we have

$$\circlearrowleft + \quad (M_R)_o = \Sigma Fd;$$

$$(M_R)_o = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$

$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

$$(M_R)_o = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m} \circlearrowright$$

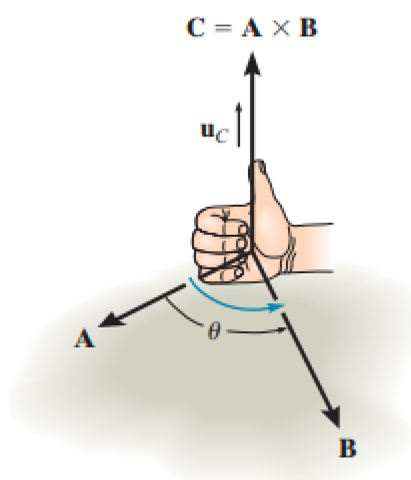
Cross Product:

The moment of a force will be formulated using Cartesian vectors in the next section. Before doing this, however, it is first necessary to expand our knowledge of vector algebra and introduce the *cross-product* method of vector multiplication. The cross product of two vectors A and B yields the vector **C**, which is written

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

and is read " C equals A cross B."

Magnitude. The magnitude of **C** is defined as the product of the magnitudes of A and B and the sine of the angle u between their tails ($0 \le \theta \le 180$). Thus, $C = AB \sin \theta$.



Direction. Vector C has a direction that is perpendicular to the plane containing A and B such that C is specified by the right-hand rule; i.e., curling the fingers of the right hand from vector A (cross) to vector B , the thumb points in the direction of C , as shown in Figure .

Moment of a Force:

The moment of a force **F** about point **O**, or actually about the moment axis passing through **O** and perpendicular to the plane containing **O** and **F**, can be expressed using the vector cross product, namely,

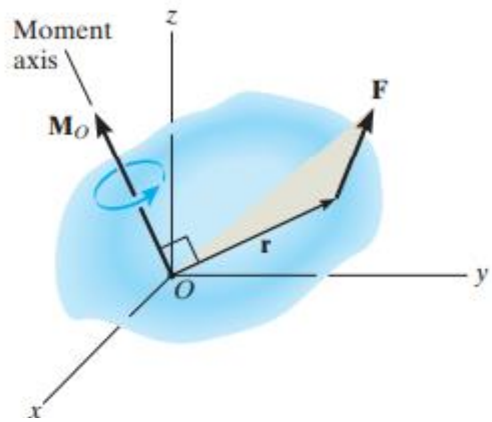
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Here r represents a position vector directed from O to any point on the line of action of F. We will now show that indeed the moment MO, when determined by this cross product, has the proper magnitude and direction. Magnitude. The magnitude of the cross product is defined as $M_O = r.F \sin \theta$, where the angle θ is measured between the tails of **r** and F.

To establish this angle, r must be treated as a sliding vector so that θ can be constructed properly. Since the moment arm d = r sin θ, then

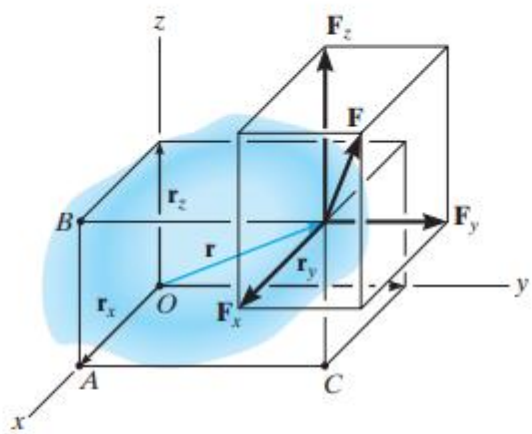
$$M_O = rF\sin\theta = F(r\sin\theta) = Fd$$

Cartesian Vector Formulation. If we establish x, y, z coordinate axes, then the position vector **r** and force **F** can be expressed as Cartesian vectors



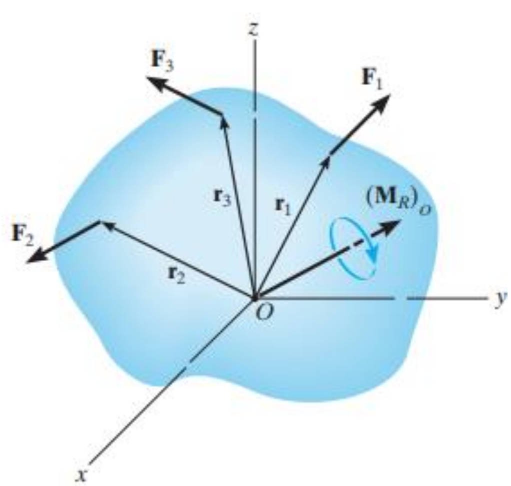
$$\boxed{\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}}$$

where r_x, r_y, r_z represent the x, y, z components of the position vector drawn from point O to any point on the line of action of the force F_x, F_y, F_z represent the x, y, z components of the force vector If the determinant is expanded, Thus, we have



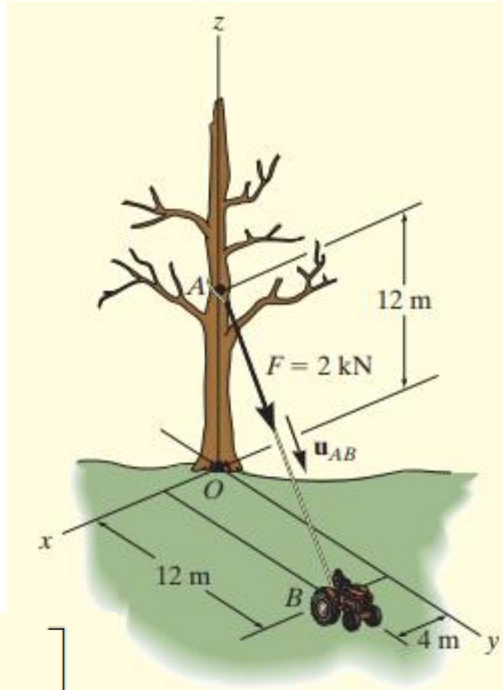
$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

Resultant Moment of a System of Forces. If a body is acted upon by a system of forces, the resultant moment of the forces about point **O** can be determined by vector addition of the moment of each force. This resultant can be written symbolically as



$$(\mathbf{M}_R)_o = \Sigma(\mathbf{r}\times\mathbf{F})$$

Example: Determine the moment produced by the force F in Fig. 4–14 a about point O . Express the result as a Cartesian vector.



Solution:

As shown, either r_A or r_B can be used to determine the moment about point **O**. These position vectors are:

$$\mathbf{r}_A = \{12\mathbf{k}\}\text{ m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\text{ m}$$

Force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\text{ kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\text{ m}}{\sqrt{(4\text{ m})^2 + (12\text{ m})^2 + (-12\text{ m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\text{ kN}$$

Thus:

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN}\cdot\text{m}$$

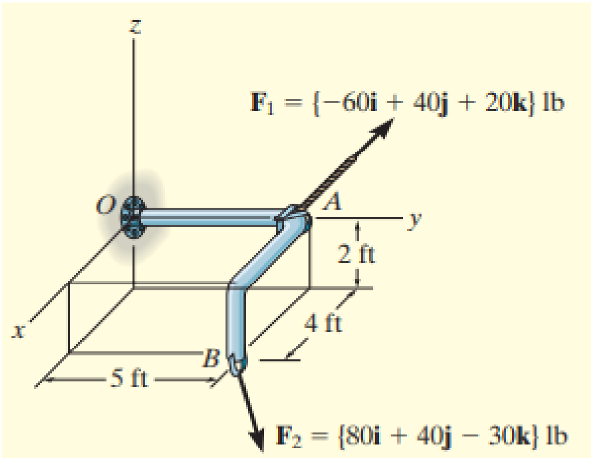
Or

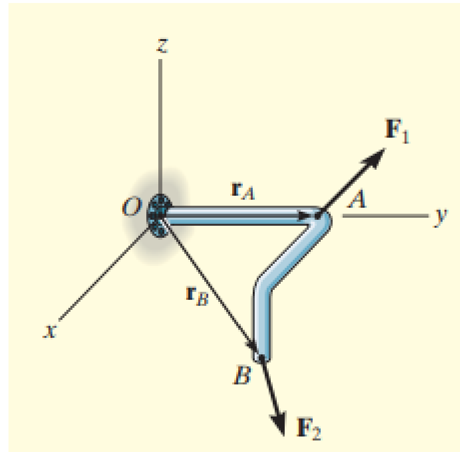
$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN}\cdot\text{m}$$

Example: Two forces act on the rod shown in Figure. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.

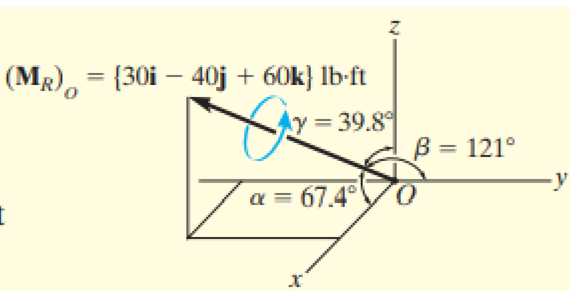




Solution:

Position vectors are directed from point O to each force as shown in Figure. These vectors are:

$$\mathbf{r}_A = \{5\mathbf{j}\}\text{ ft} \qquad \mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$



The resultant moment about O is therefore:

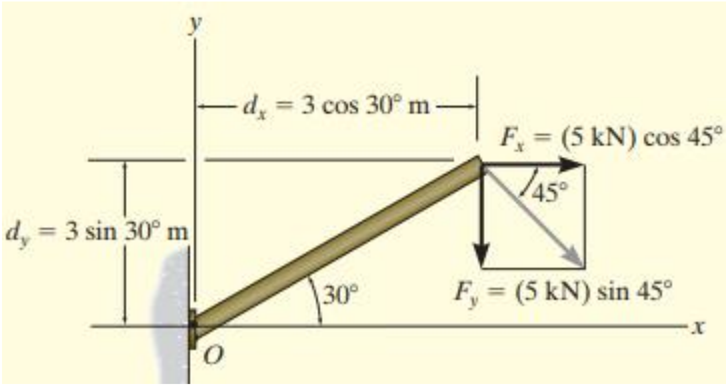
$$(\mathbf{M}_R)_o = \Sigma(\mathbf{r} \times \mathbf{F}) \quad = \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

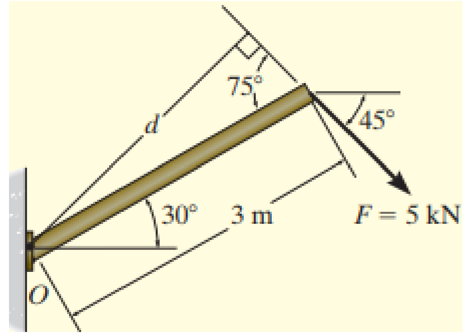
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}$$
$$+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}$$

$$= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

Example: Determine the moment of the force as shown in Figure about point O.





Solution: The moment arm **d** in can be found from trigonometry.

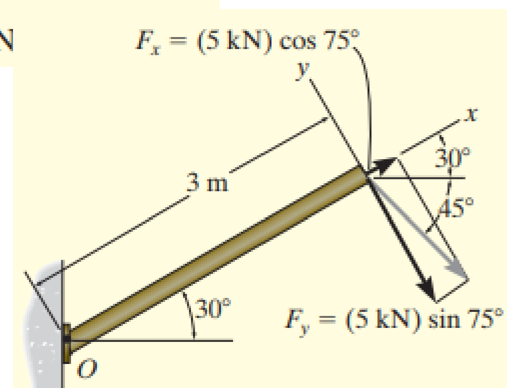
$$d = (3 \text{ m}) \sin 75° = 2.898 \text{ m}$$

Thus, $M_O = Fd = (5 \text{ kN})(2.898 \text{ m}) = 14.5 \text{ kN} \cdot \text{m} \circlearrowright$

SOLUTION II: The x and y components of the force. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$
$$= -(5 \cos 45° \text{ kN})(3 \sin 30° \text{ m}) - (5 \sin 45° \text{ kN}$$
$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright$$

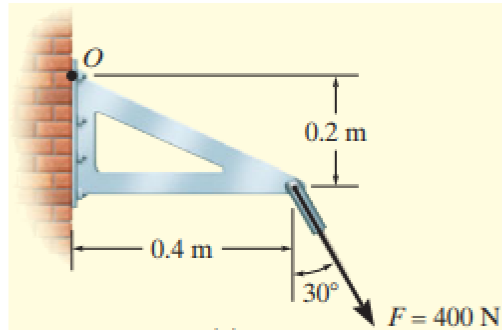
SOLUTION III: The x and y axes can be set parallel and perpendicular to the rod's axis as shown. Here F_x produces no moment about point O since its line of action passes through this point. Therefore,



(c)

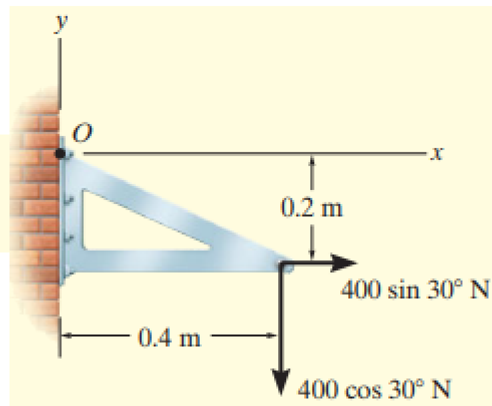
$$\circlearrowleft + M_O = -F_y d_x$$
$$= -(5 \sin 75° \text{ kN})(3 \text{ m})$$
$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright$$

Example: Force F acts at the end of the angle bracket. Determine the moment of the force about point O.



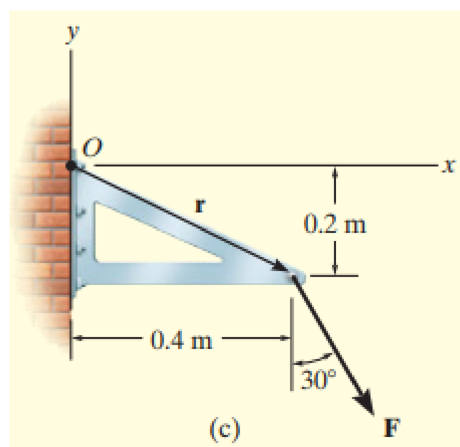
SOLUTION I (SCALAR ANALYSIS): The force is resolved into its x and y components, then

$$\circlearrowleft + M_O = 400 \sin 30^\circ \text{ N}(0.2 \text{ m}) - 400 \cos 30^\circ \text{ N}(0.4 \text{ m})$$
$$= -98.6 \text{ N}\cdot\text{m} = 98.6 \text{ N}\cdot\text{m} \circlearrowright$$



SOLUTION II (VECTOR ANALYSIS) Using a Cartesian vector approach, the force and position vectors, are:

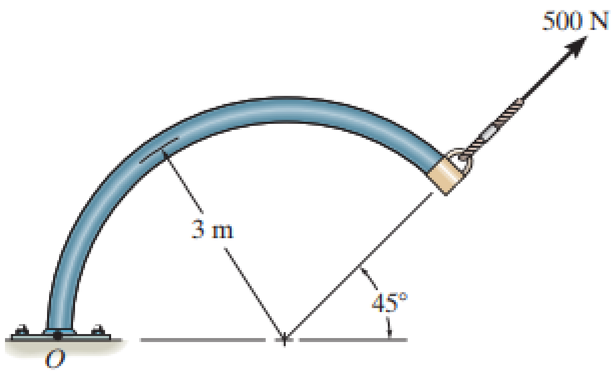
$$\mathbf{r} = \{0.4\mathbf{i} - 0.2\mathbf{j}\} \text{ m}$$
$$\mathbf{F} = \{400 \sin 30^\circ \mathbf{i} - 400 \cos 30^\circ \mathbf{j}\} \text{ N}$$
$$= \{200.0\mathbf{i} - 346.4\mathbf{j}\} \text{ N}$$

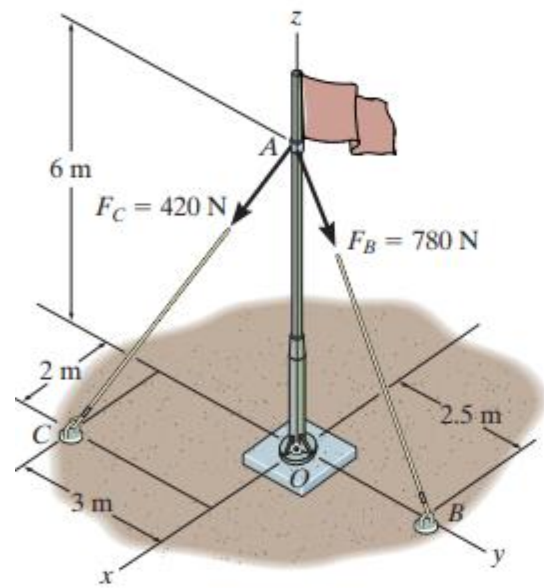


The moment is therefore:

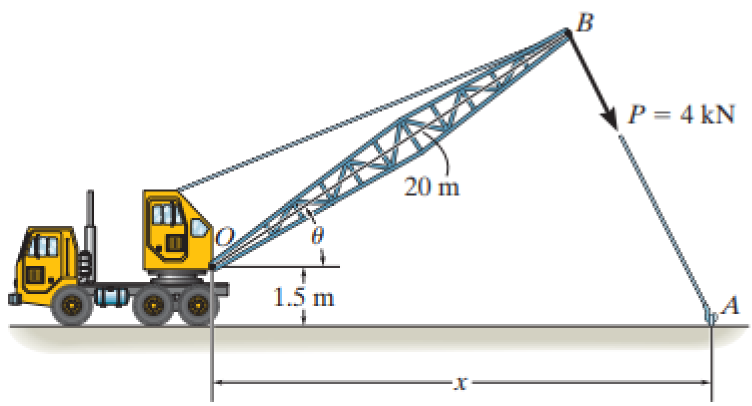
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & -0.2 & 0 \\ 200.0 & -346.4 & 0 \end{vmatrix}$$
$$= 0\mathbf{i} - 0\mathbf{j} + [0.4(-346.4) - (-0.2)(200.0)]\mathbf{k}$$
$$= \{-98.6\mathbf{k}\} \text{ N}\cdot\text{m}$$

H.W: Determine the moment of the force about point O in all figures.

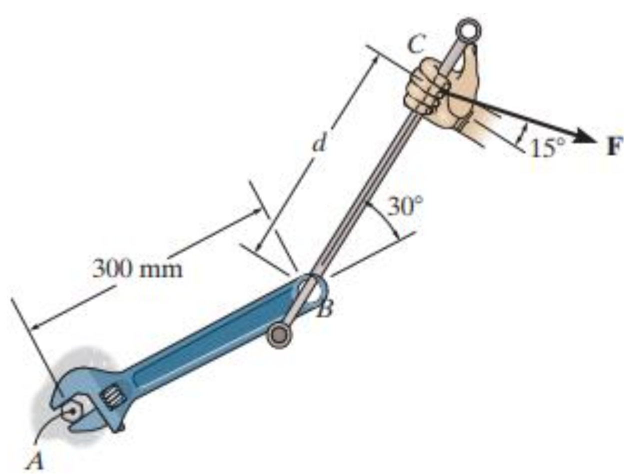




2. The towline exerts a force of P = 4 kN at the end of the 20-m-long crane boom. If x = 25 m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?

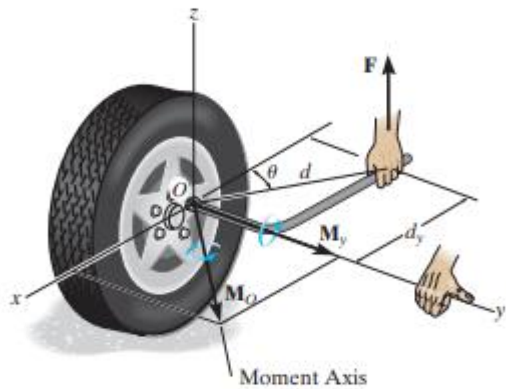


3. The connected bar BC is used to increase the lever arm of the crescent wrench as shown. If a clockwise moment of MA = 120 N . m is needed to tighten the nut at **A,** and the extension d = 300 mm, determine the required force **F** in order to develop this moment.



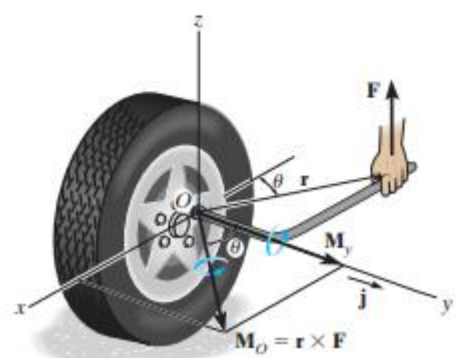
Moment of a Force about a Specified Axis:

Scalar Analysis. To use a scalar analysis in the case of the lug nut as shown in the figure., the moment arm perpendicular distance from the axis to the line of action of the force is $d_y = d \cos \theta$. Thus, the moment of F about the y axis is $M_y = F.\, d_y = F.\, (d \cos \theta)$. According to the right-hand rule, M_y is directed along the positive y axis as shown in the figure. In general, for any axis **a**, the moment is



$$M_a = F.d_a$$

Vector Analysis. To find the moment of force **F** sown about the **y** axis using a vector analysis, we must first determine the moment of the force about any point **O** on the y axis by applying the equation , $M_O = r \times F$. The component M_y along the y axis is the projection of MO onto the **y** axis. It can be found using the dot product, so that $M_y = j \,.\, M_O = j \,.\, (r * F)$, where **j** is the unit vector for the y axis. We can generalize this approach by letting u_a be the unit vector that specifies the direction of the **a** axis shown in. Then the moment of F about a point O on the axis is $M_O = r \times F$, and the projection of this moment onto the a axis is $Ma = ua \,.\, (r \times F)$. This combination is referred to as the scalar triple product. If the vectors are written in Cartesian form, we have:



$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$
$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

This result can also be written in the form of a determinant, making it easier to memorize

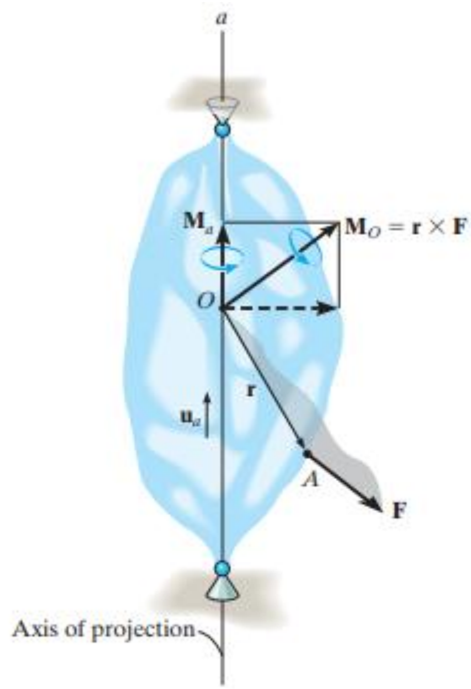
$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where:

u_{ax} , u_{ay} , u_{az} represent the x, y, z components of the unit vector defining the direction of the a axis

r_x, r_y, r_z represent the x, y, z components of the position vector extended from any point **O** on the a axis to any point A on the line of action of the force

F_x, F_y, F_z represent the x, y, z components of the force vector.



When Ma is evaluated, it will yield a positive or negative scalar. The sign of this scalar indicates the sense of direction of Ma along the **a** axis. If it is positive, then Ma will have the same sense as u_a, whereas if it is negative, then Ma will act opposite to u**a**. Once M_a is determined, we can then express M_a as a Cartesian vector, namely

$$M_a = M_a\, u_a$$

Important Points:

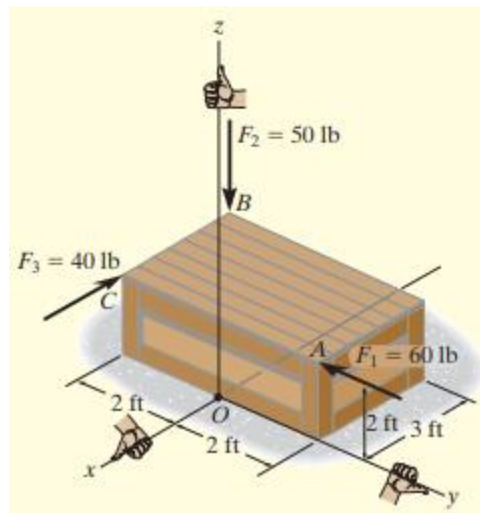
• The moment of a force about a specified axis can be determined provided the perpendicular distance da from the force line of action to the axis can be determined. $M_a = F\, d_a$.

• If vector analysis is used, $M_a = u_a \cdot (r \times F)$, where u_a defines the direction of the axis and r is extended from any point on the axis to any point on the line of action of the force.

• If Ma is calculated as a negative scalar, then the sense of direction of Ma is opposite to u_a.

• The moment Ma expressed as a Cartesian vector is determined from $M_a = M_a\, u_a$.

Example: Determine the resultant moment of the three forces shown about the x axis, the y axis, and the z axis.



Solution:

Force that is parallel to a coordinate axis or has a line of action that passes through the axis does not produce any moment or tendency for turning about that axis. Therefore, defining the positive direction of the moment of a force according to the right-hand rule, as shown in the figure, we have

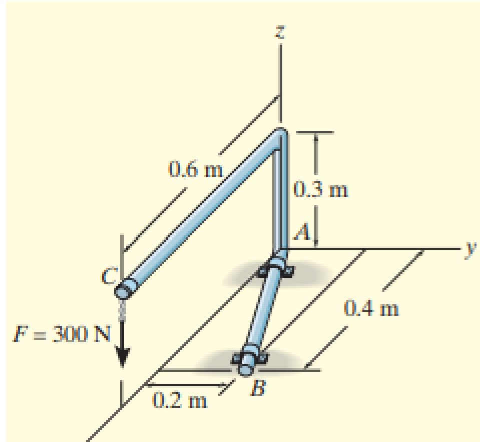
$$M_x = (60\text{ lb})(2\text{ ft}) + (50\text{ lb})(2\text{ ft}) + 0 = 220\text{ lb}\cdot\text{ft}$$

$$M_y = 0 - (50\text{ lb})(3\text{ ft}) - (40\text{ lb})(2\text{ ft}) = -230\text{ lb}\cdot\text{ft}$$

$$M_z = 0 + 0 - (40\text{ lb})(2\text{ ft}) = -80\text{ lb}\cdot\text{ft}$$

The negative signs indicate that M_y and M_z act in the -y and -z directions, respectively.

Example: Determine the moment MAB produced by the force F shown in figure, which tends to rotate the rod about the AB axis.



(a)

SOLUTION A vector analysis using $M_{AB} = u_B \cdot (r \times F)$ will be considered for the solution rather than trying to find the moment arm or perpendicular distance from the line of action of F to the AB axis. Each of the terms in the equation will now be identified. Unit vector u_B defines the direction of the AB axis of the rod, where

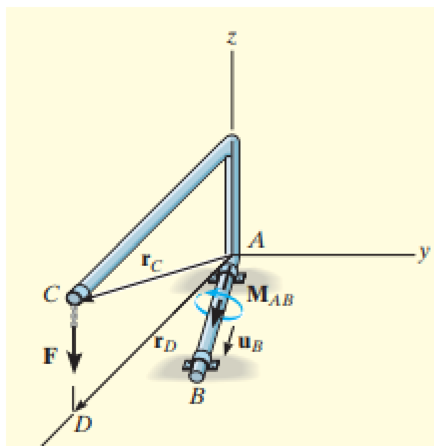
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{\{0.4\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}}{\sqrt{(0.4\ \text{m})^2 + (0.2\ \text{m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}$$

$$\mathbf{r}_D = \{0.6\mathbf{i}\}\ \text{m}$$

The force is: $$\mathbf{F} = \{-300\mathbf{k}\}\ \text{N}$$

Substituting these vectors into the determinant form and expanding, we have:

$$\begin{aligned} M_{AB} = \mathbf{u}_B \cdot (\mathbf{r}_D \times \mathbf{F}) &= \begin{vmatrix} 0.8944 & 0.4472 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix} \\ &= 0.8944[0(-300) - 0(0)] - 0.4472[0.6(-300) - 0(0)] \\ &\qquad + 0[0.6(0) - 0(0)] \\ &= 80.50\ \text{N}\cdot\text{m} \end{aligned}$$

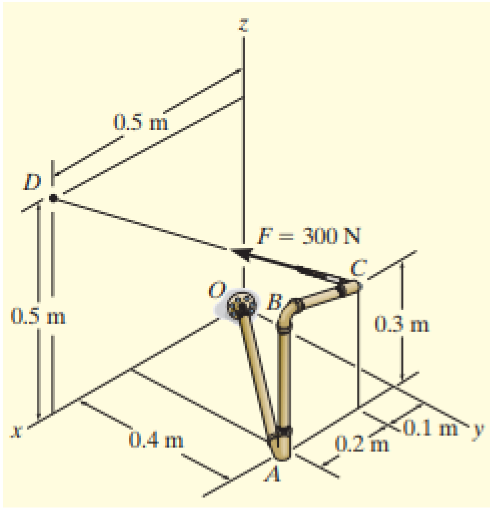


(b)

This positive result indicates that the sense of M_{AB} is in the same direction as u_B. Expressing M_{AB} as a Cartesian vector yields

$$\begin{aligned} \mathbf{M}_{AB} = M_{AB}\mathbf{u}_B &= (80.50\ \text{N}\cdot\text{m})(0.8944\mathbf{i} + 0.4472\mathbf{j}) \\ &= \{72.0\mathbf{i} + 36.0\mathbf{j}\}\ \text{N}\cdot\text{m} \end{aligned}$$

Example: Determine the magnitude of the moment of force F about segment OA of the pipe assembly as shown.



Solution:

The moment of F about the OA axis is determined from $M_{OA} = u_{OA} \cdot (r \times F)$, where r is a position vector extending from any point on the OA axis to any point on the line of action of **F**., either r_{OD}, r_{OC}, r_{AD}, or r_{AC} can be used; however, r_{OD} will be considered since it will simplify the calculation. The unit vector u_{OA}, which specifies the direction of the OA axis, is:

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{\{0.3\mathbf{i} + 0.4\mathbf{j}\}\text{ m}}{\sqrt{(0.3\text{ m})^2 + (0.4\text{ m})^2}} = 0.6\mathbf{i} + 0.8\mathbf{j}$$

and the position vector r_{OD} is:

$$\mathbf{r}_{OD} = \{0.5\mathbf{i} + 0.5\mathbf{k}\}\text{ m}$$

The force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right)$$

$$= (300\text{ N})\left[\frac{\{0.4\mathbf{i} - 0.4\mathbf{j} + 0.2\mathbf{k}\}\text{ m}}{\sqrt{(0.4\text{ m})^2 + (-0.4\text{ m})^2 + (0.2\text{ m})^2}}\right]$$

$$= \{200\mathbf{i} - 200\mathbf{j} + 100\mathbf{k}\}\text{ N}$$

Therefore,

$$M_{OA} = \mathbf{u}_{OA} \cdot (\mathbf{r}_{OD} \times \mathbf{F})$$

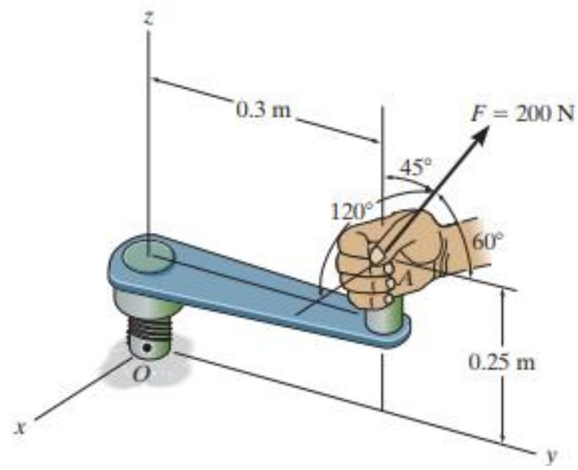
$$= \begin{vmatrix} 0.6 & 0.8 & 0 \\ 0.5 & 0 & 0.5 \\ 200 & -200 & 100 \end{vmatrix}$$

$$= 0.6[0(100) - (0.5)(-200)] - 0.8[0.5(100) - (0.5)(200)] + 0$$

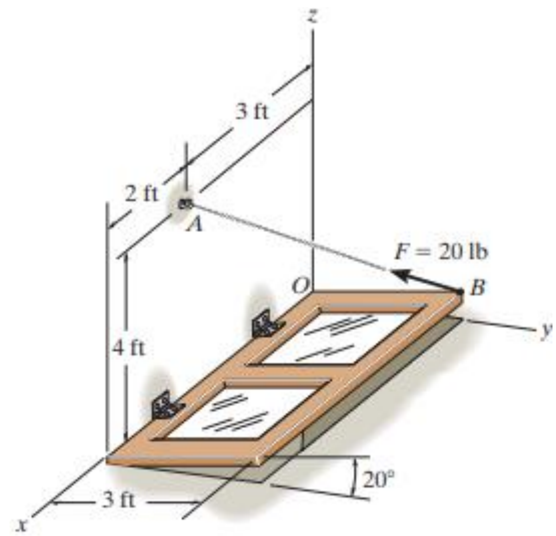
$$= 100\text{ N}\cdot\text{m}$$

H.W

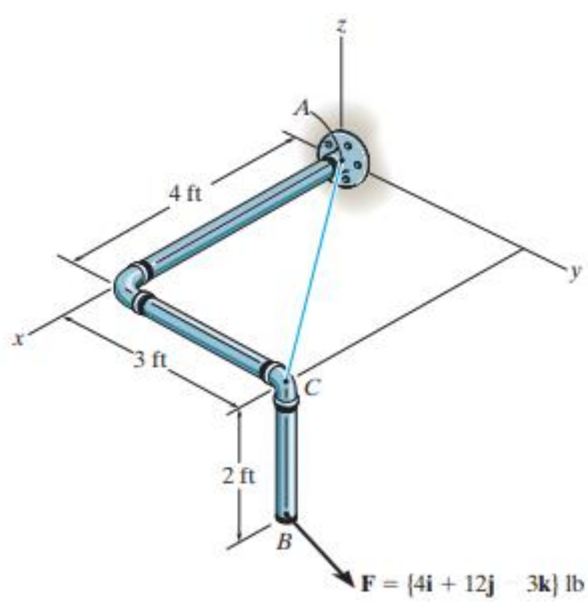
1. Determine the magnitude of the moment of the 200-N force about the x axis. Solve the problem using both a scalar and a vector analysis.



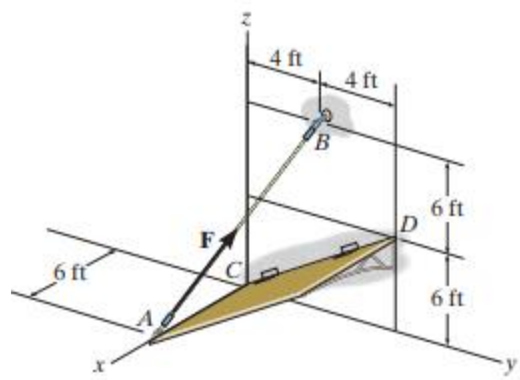
2. The chain AB exerts a force of 20 lb on the door at B. Determine the magnitude of the moment of this force along the hinged axis x of the door.



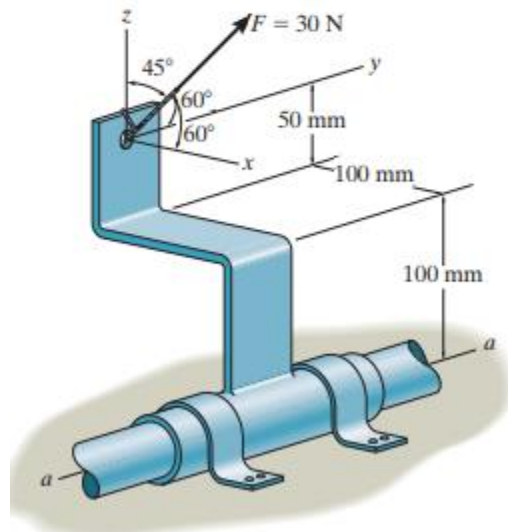
3. Determine the moment of the force F about an axis extending between A and C. Express the result as a Cartesian vector.



4. Determine the magnitude of force F in cable AB in order to produce a moment of 500 lb. ft about the hinged axis CD, which is needed to hold the panel in the position shown.

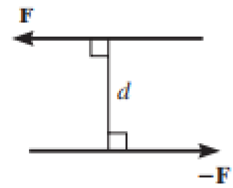


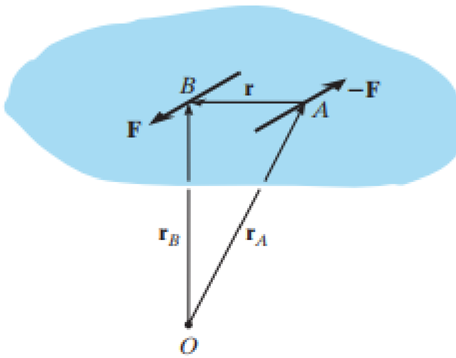
5. The force of F = 30 N acts on the bracket as shown. Determine the moment of the force about the a-a axis of the pipe. Also, determine the coordinate direction angles of F in order to produce the maximum moment about the a-a axis. What is this moment?



Moment of a Couple:

A couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, Since the resultant force is zero, the only effect of a couple is to produce an actual rotation, or if no movement is possible, there is a tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate. The moment produced by a couple is called a couple moment. We can determine its value by finding the sum of the moments of both couple forces about any arbitrary point. For example, position vectors r_A and r_B are directed from point O to points A and B lying on the line of action of -**F** and **F**. The couple moment determined about O is therefore





$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

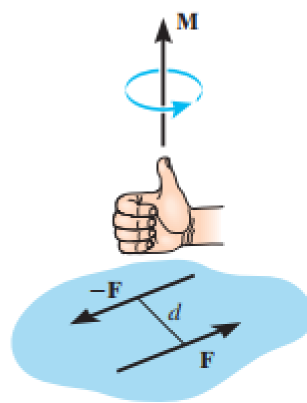
However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Scalar Formulation. The moment of a couple, M, is defined as having a magnitude of

$$\boxed{M = Fd}$$

where F is the magnitude of one of the forces and d is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right-hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces. In all cases, M will act perpendicular to the plane containing these forces.



Vector Formulation. The moment of a couple can also be expressed by the vector cross product:

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

Application of this equation is easily remembered if one thinks of taking the moments of both forces about a point lying on the line of action of one of the forces. For example, if moments are taken about point A, the moment of -F is zero about this point. Therefore, in the formulation r is crossed with the force F to which it is directed.

Equivalent Couples. If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent.

Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition. If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$

Important Points

• A couple moment is produced by two non-collinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.

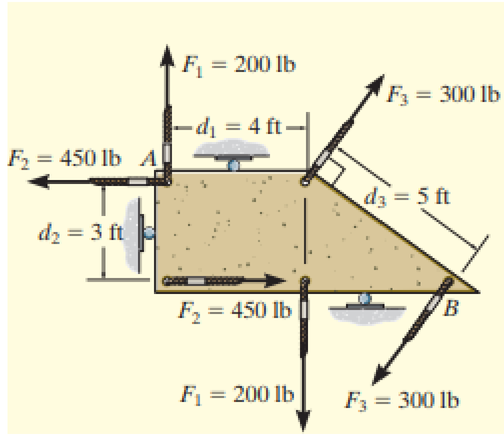
• A couple moment is a free vector, and as a result it causes the same rotational effect on a body regardless of where the couple moment is applied to the body.

• The moment of the two couple forces can be determined about any point. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.

• In three dimensions the couple moment is often determined using the vector formulation, M = r x F, where r is directed from any point on the line of action of one of the forces to any point on the line of action of the other force F.

• A resultant couple moment is simply the vector sum of all the couple moments of the system.

Example: Determine the resultant couple moment of the three couples acting on the plate in the figure



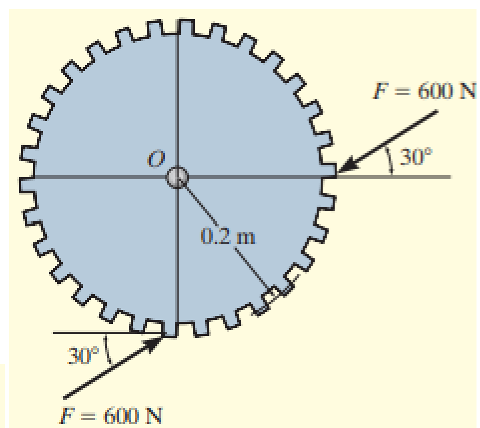
Solution:

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft + M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$
$$= -(200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

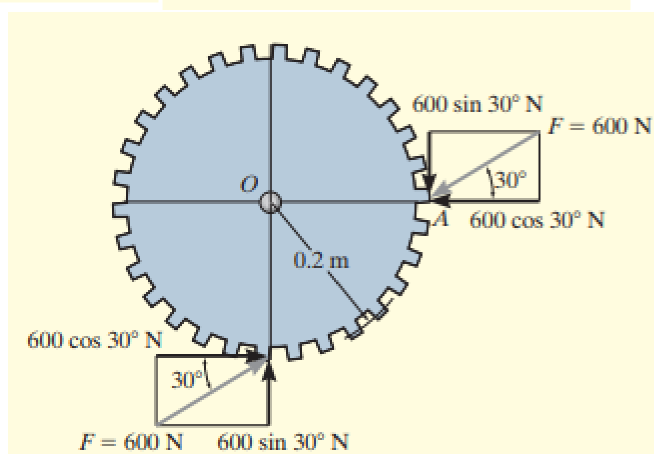
$$= -950 \text{ lb} \cdot \text{ft} = 950 \text{ lb} \cdot \text{ft} \circlearrowright$$

Example: Determine the magnitude and direction of the couple moment acting on the gear in the figure.

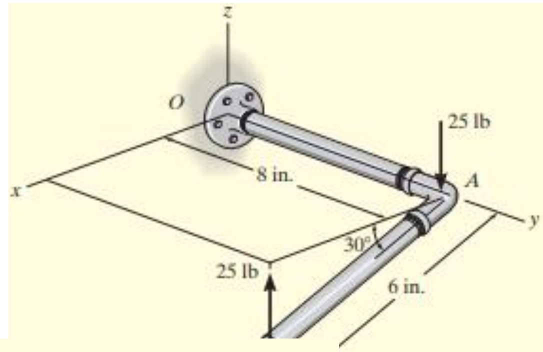


$$\circlearrowleft + M = \Sigma M_O;\ M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

$$= 43.9 \text{ N} \cdot \text{m} \circlearrowleft$$

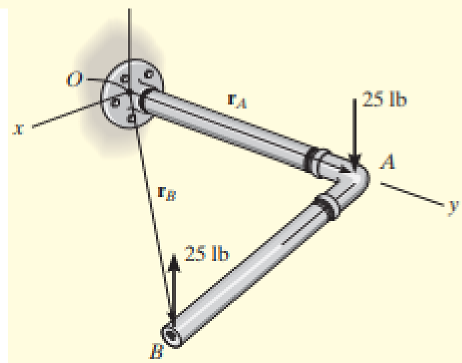


Example: Determine the couple moment acting on the pipe shown. Segment AB is directed 30° below the x–y plane.



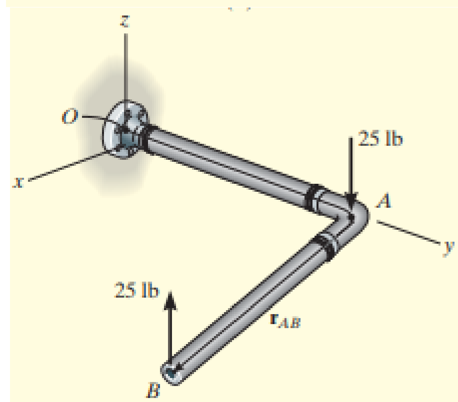
SOLUTION I (VECTOR ANALYSIS) The moment of the two couple forces can be found about any point. If point O is considered, we have:

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})\\ &= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30^\circ\mathbf{i} + 8\mathbf{j} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})\\ &= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}\\ &= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.}\end{aligned}$$

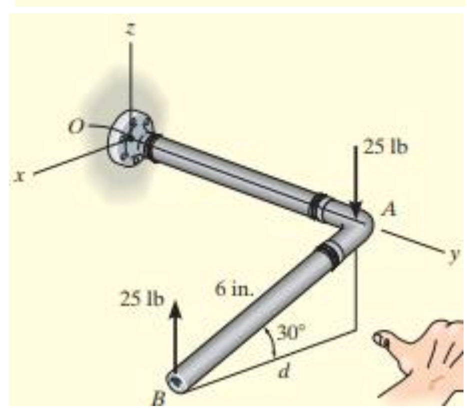


It is easier to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point A, In this case the moment of the force at A is zero, so that

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_{AB} \times (25\mathbf{k})\\ &= (6\cos 30^\circ\mathbf{i} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})\\ &= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.}\end{aligned}$$



SOLUTION II (SCALAR ANALYSIS) although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation M = F.d. The perpendicular distance between the lines of action of the couple forces is d = 6 cos 30 = 5.196 in. Hence, taking moments of the forces about either point A or point B yields

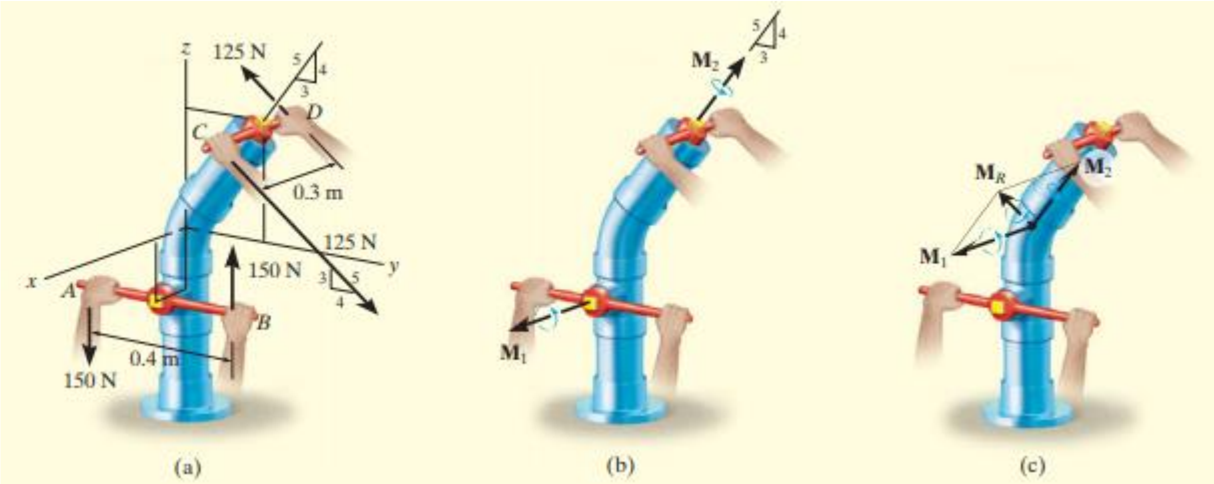


$$M = Fd = 25\ \text{lb}\,(5.196\ \text{in.}) = 129.9\ \text{lb}\cdot\text{in.}$$

Applying the right-hand rule, M acts in the -j direction. Thus,

$$\mathbf{M} = \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.}$$

Example: Replace the two couples acting on the pipe column in Fig. 4–33 a by a resultant couple moment.



SOLUTION (VECTOR ANALYSIS): The couple moment M_1, developed by the forces at A and B, can easily be determined from a scalar formulation.

$$M_1 = Fd = 150\text{ N}(0.4\text{ m}) = 60\text{ N}\cdot\text{m}$$

By the right-hand rule, M_1 acts in the +i direction, Figure b. Hence,

$$\mathbf{M}_1 = \{60\mathbf{i}\}\text{ N}\cdot\text{m}$$

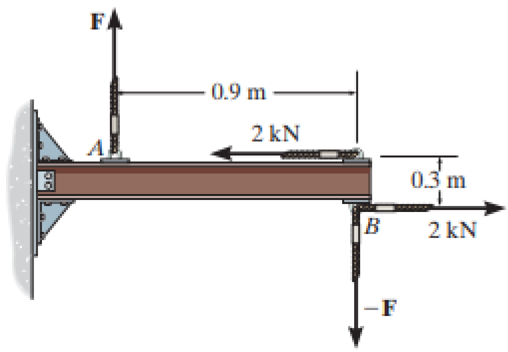
Vector analysis will be used to determine M2, caused by forces at C and D. If moments are calculated about point D, Fig. a, $M_2 = r_{DC}$ x FC, then

$$\begin{aligned}\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C &= (0.3\mathbf{i}) \times \left[125\left(\tfrac{4}{5}\right)\mathbf{j} - 125\left(\tfrac{3}{5}\right)\mathbf{k}\right]\\ &= (0.3\mathbf{i}) \times [100\mathbf{j} - 75\mathbf{k}] = 30(\mathbf{i} \times \mathbf{j}) - 22.5(\mathbf{i} \times \mathbf{k})\\ &= \{22.5\mathbf{j} + 30\mathbf{k}\}\text{ N}\cdot\text{m}\end{aligned}$$

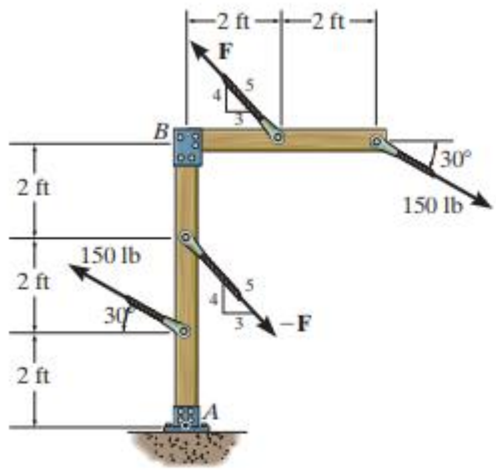
Since M_1 and M_2 are free vectors, they may be moved to some arbitrary point and added vectorially, Fig. c . The resultant couple moment becomes

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = \{60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k}\}\text{ N}\cdot\text{m}$$

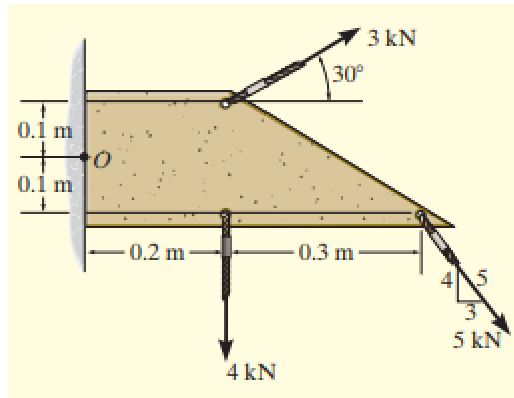
Q1/ Determine the magnitude of **F** so that the resultant couple moment acting on the beam is 1.5 kN.m clockwise.



Q2/ Determine the required magnitude of force F if the resultant couple moment on the frame is 200 lb .ft , clockwise.

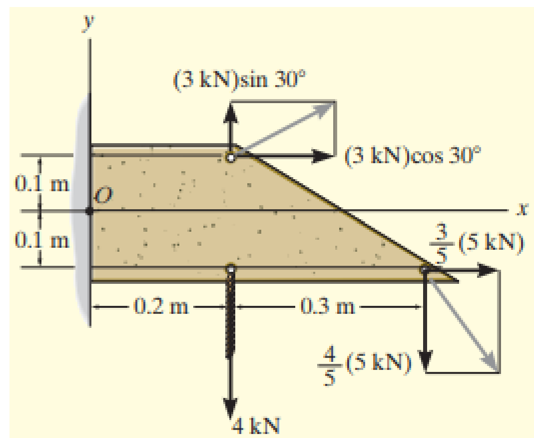


Example: Replace the force and couple system shown by an equivalent resultant force and couple moment acting at point O.



Solution:

Force Summation: The 3 kN and 5 kN forces are resolved into their x and y components as shown. We have:



$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30° + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30° - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37*c*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN}$$

Its direction θ is

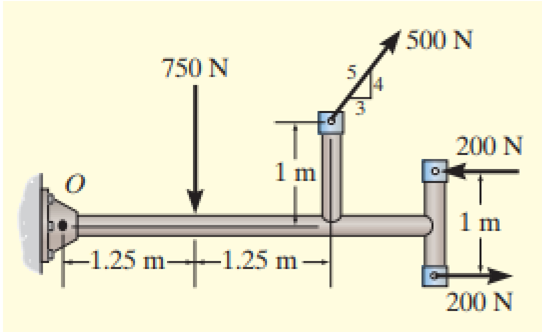
$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3°$$

Moment Summation. The moments of 3 kN and 5 kN about point O will be determined using their x and y components, we have

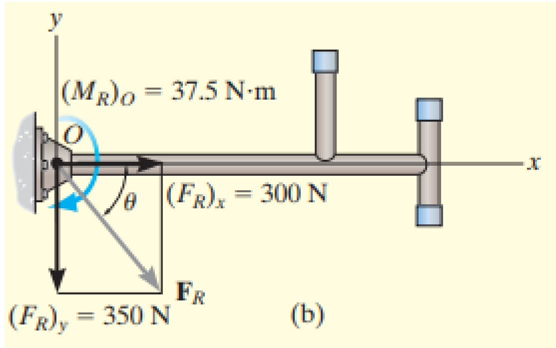
$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30°(0.2\text{ m}) - (3\text{ kN})\cos 30°(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})\,(0.1\text{ m})$$
$$- \left(\tfrac{4}{5}\right)(5\text{ kN})\,(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$
$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m}\circlearrowright$$

Example: Replace the force and couple system acting on the member shown by an equivalent resultant force and couple moment acting at point O.



Solution: Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its x and y components, thus,



$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x;\ (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\ (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

The magnitude of $\mathbf{F_R}$ is:

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ$$

Moment Summation: Since the couple moment is a free vector, it can act at any point on the member. We have

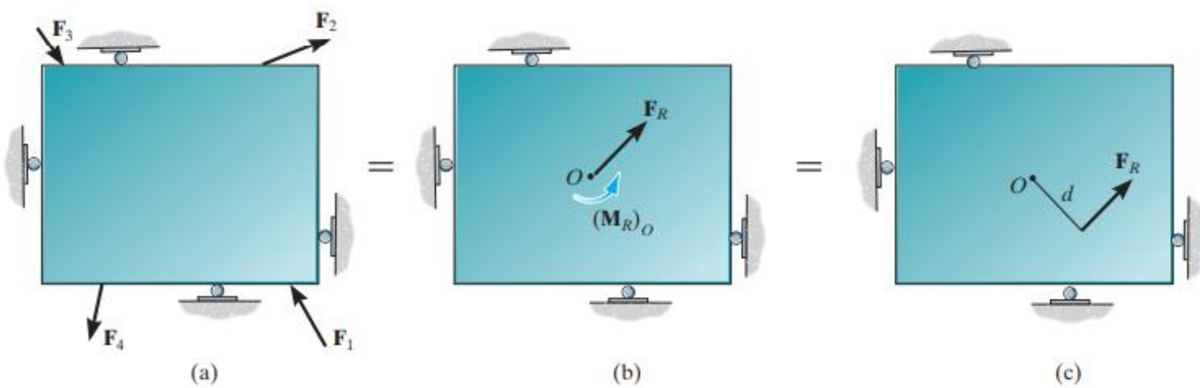
$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m}) - (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright$$

Coplanar Force System:

In the case of a coplanar force system , the lines of action of all the forces lie in the same plane as shown Figure **a**, and so the resultant force FR = ΣF of this system also lies in this plane. Furthermore, the moment of each of the forces about any point O is directed perpendicular to this plane. Thus, the resultant moment $(MR)_O$ and resultant force F_R will be mutually perpendicular , Figure **b** . The resultant moment can be replaced by moving the resultant force F_R a perpendicular or moment arm distance d away from point O such that FR produces the same moment $(MR)_O$ about point O , Figure **c** . This distance **d** can be determined from the scalar equation $(MR)_O = F_R.d = ΣMO$ or $d = (MR)_O/F_R$



Procedure for Analysis: The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

• Establish the x , y , z , axes and locate the resultant force FR an arbitrary distance away from the origin of the coordinates. Force Summation.

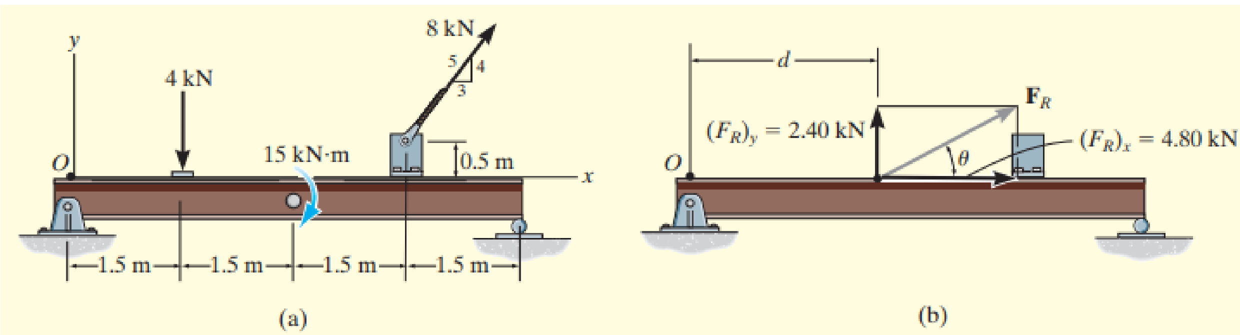
• The resultant force is equal to the sum of all the forces in the system.

• For a coplanar force system, resolve each force into its x and y components. Positive components are directed along the positive x and y axes, and negative components are directed along the negative x and y axes. Moment Summation.

• The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O . • This moment condition is used to find the location of the resultant force from point O .

Example:

Replace the force and couple moment system acting on the beam in Fig. 4–44 a by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.



Solution:

Force Summation. Summing the force components,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ$$

Moment Summation. We must equate the moment of **F_R** about point O in Fig. b to the sum of the moments of the force and couple moment system about point O in Fig. a . Since the line of action of **$(F_R)_x$** acts through point O , only **$(F_R)_y$** produces a moment about this point. Thus,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m}) \qquad d = 2.25\text{ m}$$

Equilibrium of a Rigid Body:

CHAPTER OBJECTIVES

To develop the equations of equilibrium for a rigid body.

To introduce the concept of the free-body diagram for a rigid body.

To show how to solve rigid-body equilibrium problems using the equations of equilibrium.

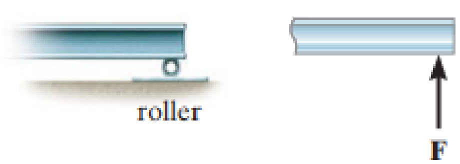
EQUILIBRIUM IN TWO DIMENSIONS:

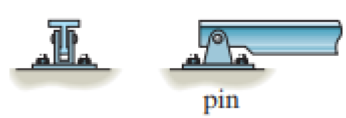
Free-Body Diagrams:

Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule,

• If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.

• If rotation is prevented, a couple moment is exerted on the body.





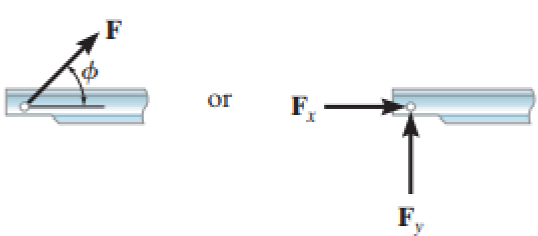
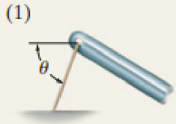
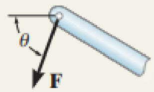
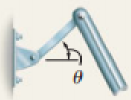
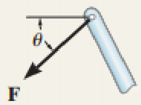
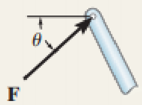
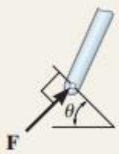
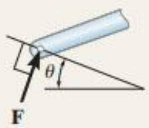
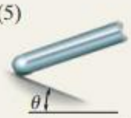
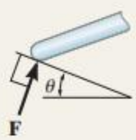
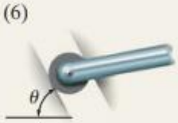
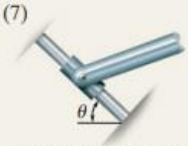
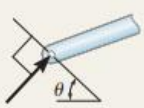
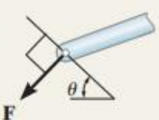
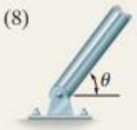
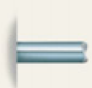
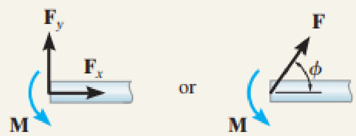


TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

continued

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Procedure for Analysis: To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

Draw Outlined Shape. Imagine the body to be isolated or cut “free” from its constraints and connections and draw (sketch) its outlined shape.

Show All Forces and Couple Moments. Identify all the known and unknown external forces and couple moments that act on the body. Those generally encountered are due to (1) applied loadings, (2) reactions occurring at the supports or at points of contact with other bodies (see Table 5–1), and (3) the weight of the body. To account for all these effects, it may help to trace over the boundary, carefully noting each force or couple moment acting on it.

Identify Each Loading and Give Dimensions. The forces and couple moments that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couple moments that are unknown. Establish an x, y coordinate system so that these unknowns, Ax, Ay, etc., can be identified. Finally, indicate the dimensions of the body necessary for calculating the moments of forces.

Important Points

• No equilibrium problem should be solved without first drawing the free-body diagram, so as to account for all the forces and couple moments that act on the body.

• If a support prevents translation of a body in a particular direction, then the support exerts a force on the body in that direction.

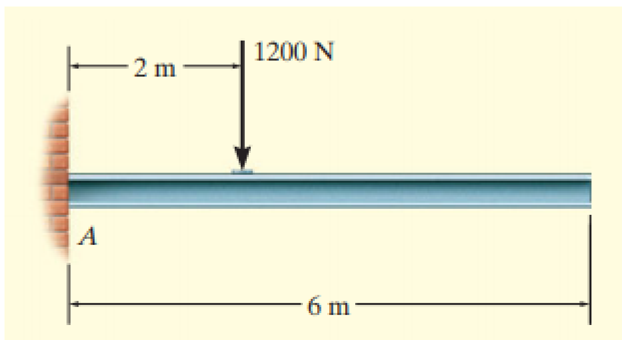
• If rotation is prevented, then the support exerts a couple moment on the body. • Study Table 5–1.

• Internal forces are never shown on the free-body diagram since they occur in equal but opposite collinear pairs and therefore cancel out.

• The weight of a body is an external force, and its effect is represented by a single resultant force acting through the body's center of gravity G.

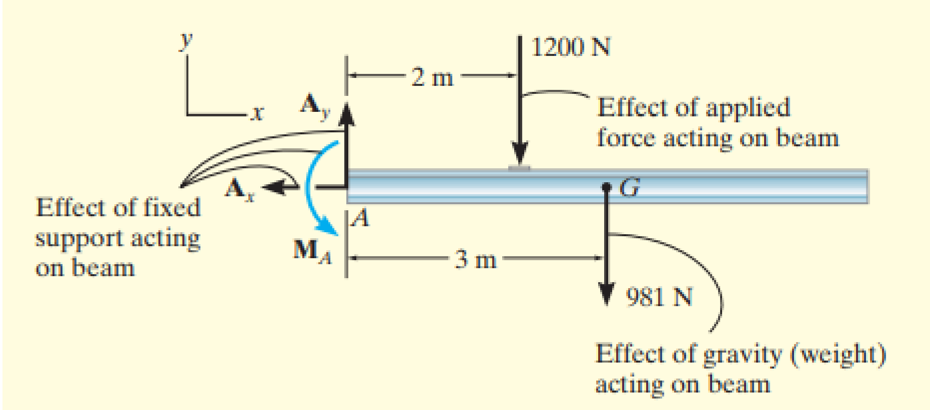
• Couple moments can be placed anywhere on the free-body diagram since they are free vectors. Forces can act at any point along their lines of action since they are sliding vectors.

Example: Draw the free-body diagram of the uniform beam shown in Figure a. The beam has a mass of 100 kg.



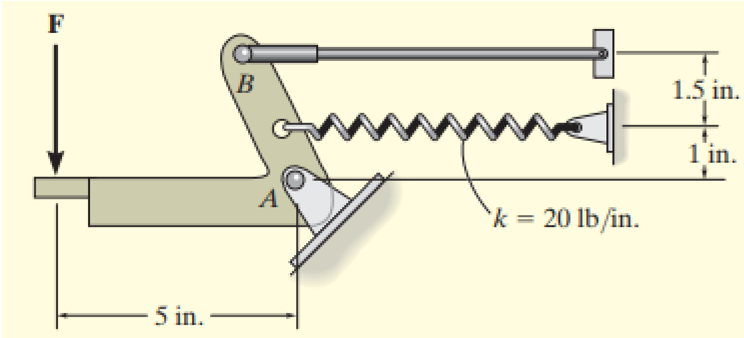
(a)

Solution: SOLUTION The free-body diagram of the beam is shown in Figure b. Since the support at A is fixed, the wall exerts three reactions on the beam, denoted as Ax, Ay, and M_A. The magnitudes of these reactions are unknown, and their sense has been assumed. The weight of the beam, W = 100(9.81) N = 981 N, acts through the beam's center of gravity G, which is 3 m from A since the beam is uniform.



(b)

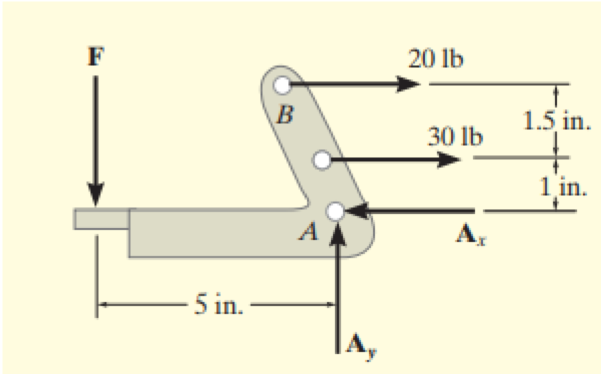
Example: Draw the free-body diagram of the foot lever shown in Figure a. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force on the link at B is 20 lb.



(b)

Solution:

By inspection of the photo the lever is loosely bolted to the frame at A and so this bolt acts as a pin. (See (8) in Table 5–1.) Although not shown here the link at B is pinned at both ends and so it is like (2) in Table 5-1. After making the proper measurements, the idealized model of the lever is shown in Fig. c . From this, the free-body diagram is shown in Fig. 5–8 c.

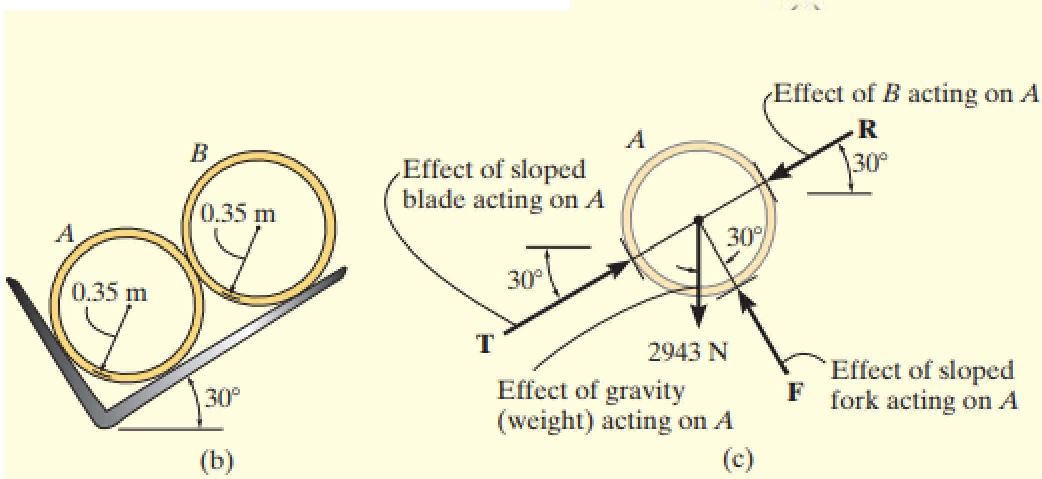


(c)

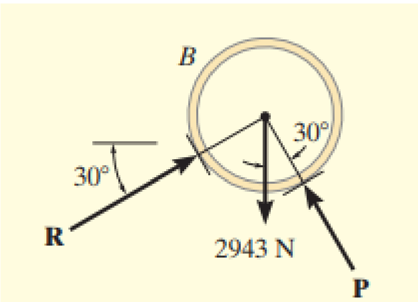
The pin at A exerts force components Ax and Ay on the lever. The link exerts a force of 20 lb, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be k = 20 lb.in., then since the stretch s = 1.5 in., Fs = k.s = 20 lb>in. (1.5 in.) = 30 lb. Finally, the operator's shoe applies a vertical force of F on the pedal. The dimensions of the lever are also shown on the free-body diagram, since this information will be useful when calculating the moments of the forces. As usual, the senses of the unknown forces at A have been assumed. The correct senses will become apparent after solving the equilibrium equations.

Example: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Figure a. Draw the free-body diagrams for each pipe and both pipes together.

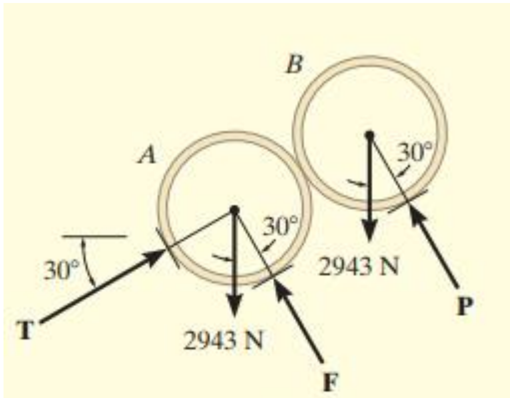
Solution:



The idealized model from which we must draw the free-body diagrams is shown in Figure b. Here the pipes are identified, the dimensions have been added, and the physical situation reduced to its simplest form. The free-body diagram for pipe A is shown in Fig. 5–9 c . Its weight is W = 300(9.81) N = 2943 N. Assuming all contacting surfaces are smooth , the reactive forces **T** , **F** , **R** act in a direction normal to the tangent at their surfaces of contact. The free-body diagram of pipe B is shown in Figure d . Can you identify each of the three forces acting on this pipe ? In particular, note that **R** , representing the force of A on B , Figured , is equal and opposite to **R** representing the force of B on A , Figure c . This is a consequence of Newton's third law of motion. The free-body diagram of both pipes combined ("system") is shown in Figure e. Here the contact force **R**, which acts between A and B, is considered as an internal force and hence is not shown on the free-body diagram. That is, it represents a pair of equal but opposite collinear forces which cancel each other.



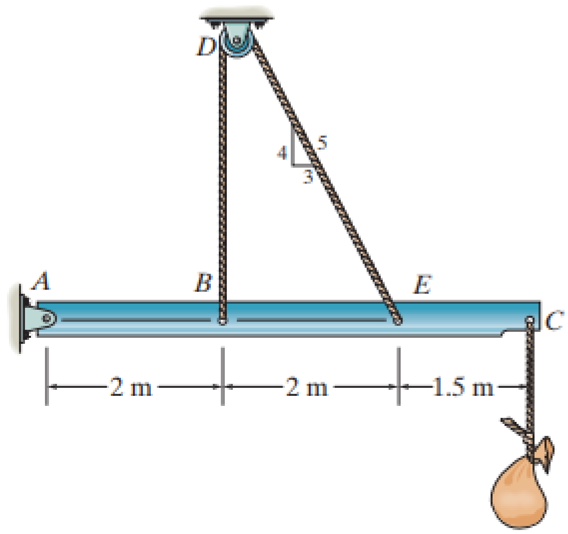
(d)



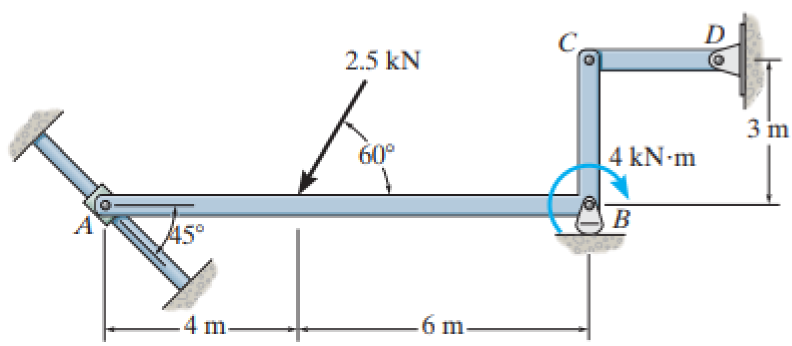
(e)

H.W:

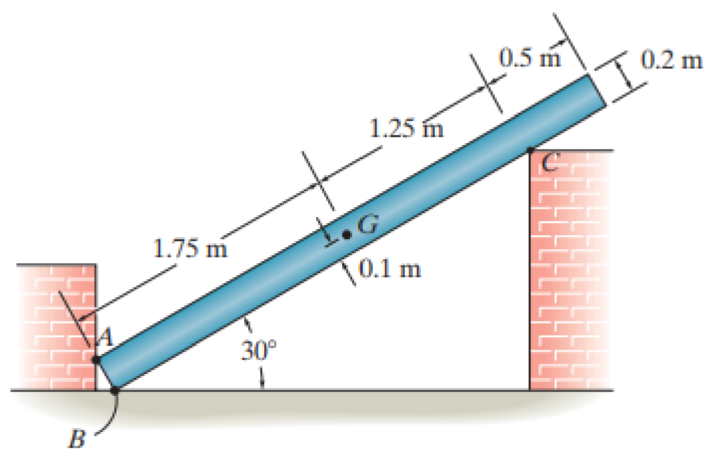
1. Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram



2. Draw the free-body diagram of member ABC which is supported by a smooth collar at A, rocker at B, and short link CD . Explain the significance of each force acting on the diagram.



3. Draw the free-body diagram of the uniform bar, which has a mass of 100 kg and a center of mass at G. The supports A, B, and C are smooth.

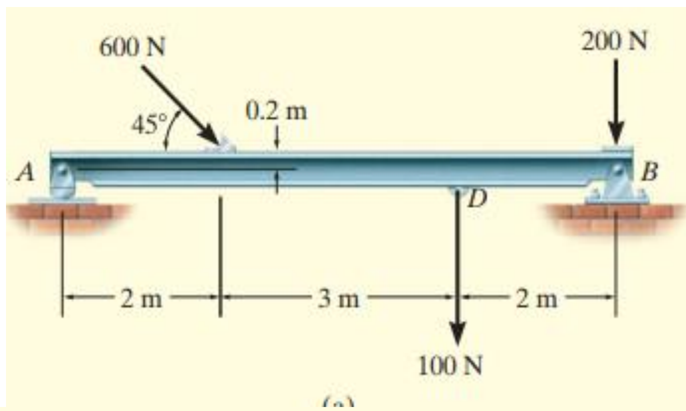


Equations of Equilibrium:

In previous section, we developed the two equations which are both necessary and sufficient for the equilibrium of a rigid body, namely, ΣF = 0 and ΣMO = 0. When the body is subjected to a system of forces, which all lie in the x – y plane, then the forces can be resolved into their x and y components. Consequently, the conditions for equilibrium in two dimensions are:

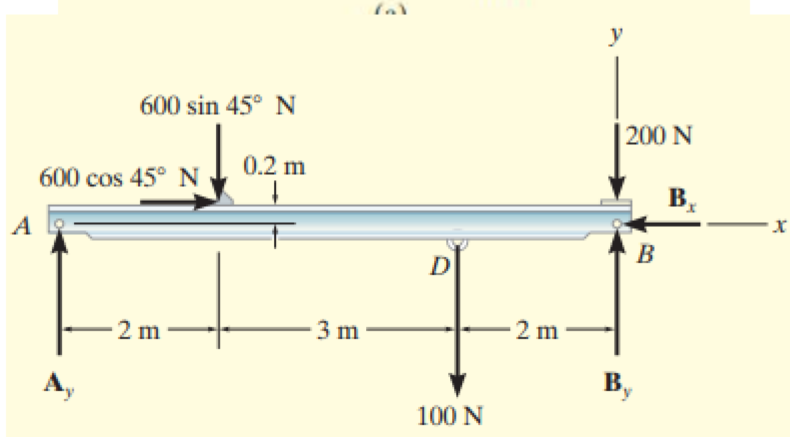
$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned}}$$

Example: Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown. Neglect the weight of the beam.



Solution:

Free-Body Diagram. Identify each of the forces shown on the free-body diagram of the beam, Figure b, for simplicity, the 600-N force is represented by its x and y components as shown in Figure b



(b)

Equations of Equilibrium. Summing forces in the x direction yields:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N}$$

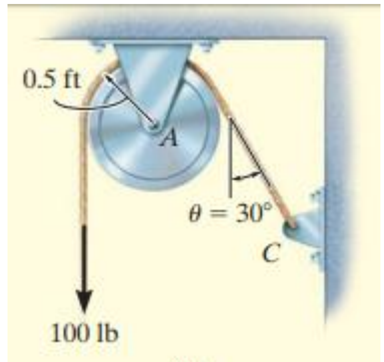
A direct solution for A_y can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B:

$$\circlearrowleft + \Sigma M_B = 0; \quad 100\text{ N}(2\text{ m}) + (600 \sin 45^\circ \text{ N})(5\text{ m}) - (600 \cos 45^\circ \text{ N})(0.2\text{ m}) - A_y(7\text{ m}) = 0$$
$$A_y = 319\text{ N}$$

Summing forces in the y direction, using this result, gives:

$$+\uparrow \Sigma F_y = 0; \quad 319\text{ N} - 600 \sin 45^\circ \text{ N} - 100\text{ N} - 200\text{ N} + B_y = 0$$
$$B_y = 405\text{ N}$$

Example: The cord shown in Figure **a** supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



(a)

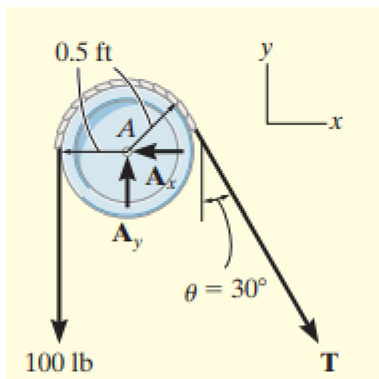
Solution: Equations of Equilibrium. Summing moments about point A to eliminate A_x and Ay, Figure c , we have:

$$\circlearrowleft + \Sigma M_A = 0; \quad 100\text{ lb}(0.5\text{ ft}) - T(0.5\text{ ft}) = 0$$
$$T = 100\text{ lb}$$

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 100 \sin 30^\circ \text{ lb} = 0$$
$$A_x = 50.0\text{ lb}$$

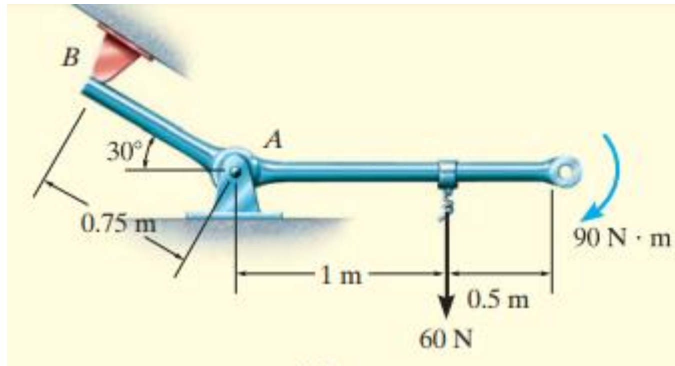
$$+\uparrow \Sigma F_y = 0; \quad A_y - 100\text{ lb} - 100 \cos 30^\circ \text{ lb} = 0$$
$$A_y = 187\text{ lb}$$



(c)

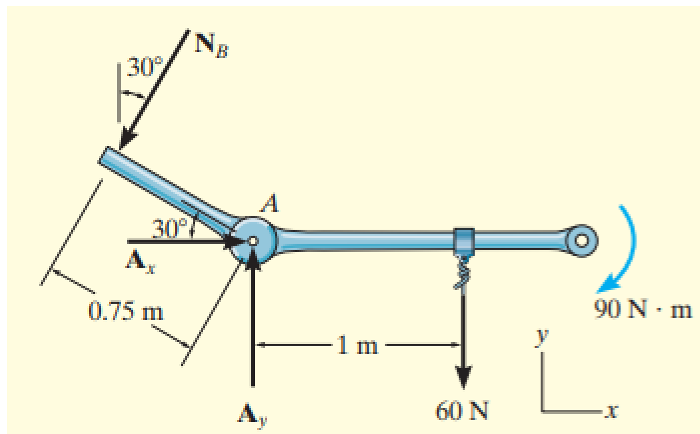
Example: The member shown in Figure **a** is pin connected at **A** and rests against a smooth support at **B** . Determine the horizontal and vertical components of reaction at the pin **A.**

Solution:



(a)

Free-Body Diagram. As shown in Figure **b** the reaction $\mathbf{N_B}$ is perpendicular to the member at **B**. Also, horizontal and vertical components of reaction are represented at **A**.



(b)

Equations of Equilibrium. Summing moments about **A**, we obtain a direct solution for NB,

$$\circlearrowleft +\Sigma M_A = 0; \quad -90\ \text{N}\cdot\text{m} - 60\ \text{N}(1\ \text{m}) + N_B(0.75\ \text{m}) = 0$$

$$N_B = 200\ \text{N}$$

Using this result,

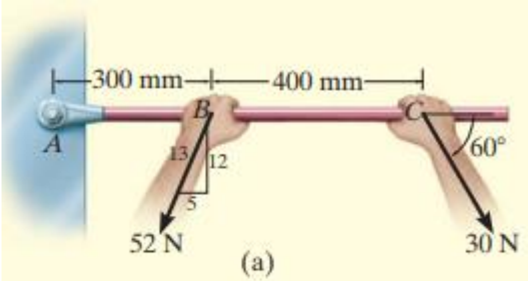
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 200 \sin 30^\circ\ \text{N} = 0$$

$$A_x = 100\ \text{N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200 \cos 30^\circ\ \text{N} - 60\ \text{N} = 0$$

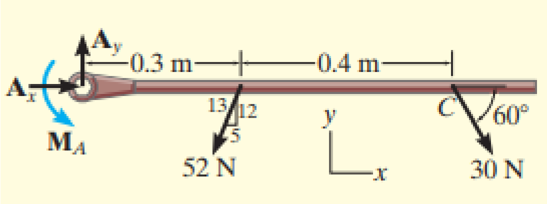
$$A_y = 233\ \text{N}$$

Example: The box wrench in Figure a is used to tighten the bolt at **A.** If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to the bolt and the force of the wrench on the bolt.



(a)

Solution: Free-Body Diagram. The free-body diagram for the wrench is shown in Figure b. Since the bolt acts as a "fixed support," it exerts force components $\mathbf{A_x}$ and Ay and a moment $\mathbf{M_A}$ on the wrench at **A**.



(b)

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 52\left(\tfrac{5}{13}\right) \text{N} + 30 \cos 60^\circ \text{ N} = 0$$
$$A_x = 5.00 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 52\left(\tfrac{12}{13}\right) \text{N} - 30 \sin 60^\circ \text{ N} = 0$$
$$A_y = 74.0 \text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad M_A - \left[52\left(\tfrac{12}{13}\right)\text{N}\right](0.3 \text{ m}) - (30 \sin 60^\circ \text{ N})(0.7 \text{ m}) = 0$$
$$M_A = 32.6 \text{ N}\cdot\text{m}$$

$$F_A = \sqrt{(5.00)^2 + (74.0)^2} = 74.1 \text{ N}$$

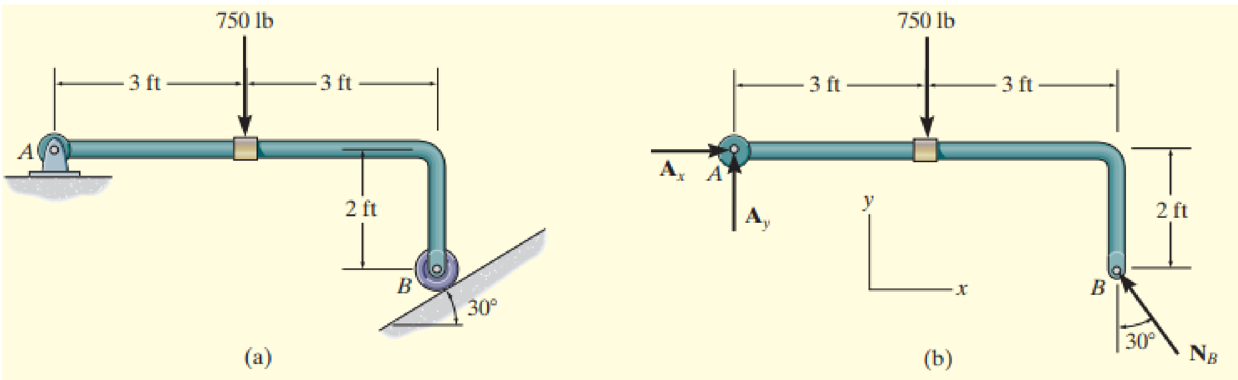
Checking:

$$\circlearrowleft + \Sigma M_C = 0; \quad \left[52\left(\tfrac{12}{13}\right)\text{N}\right](0.4 \text{ m}) + 32.6 \text{ N}\cdot\text{m} - 74.0 \text{ N}(0.7 \text{ m}) = 0$$
$$19.2 \text{ N}\cdot\text{m} + 32.6 \text{ N}\cdot\text{m} - 51.8 \text{ N}\cdot\text{m} = 0$$

Example: Determine the horizontal and vertical components of reaction on the member at the pin **A**, and the normal reaction at the roller **B** in Figure a.

Solution:

Free-Body Diagram. The free-body diagram is shown in Figure b. The pin at **A** exerts two components of reaction on the member, $\mathbf{A_x}$ and $\mathbf{A_y}$.



Equations of Equilibrium. The reaction $\mathbf{N_B}$ can be obtained directly by summing moments about point **A**, since $\mathbf{A_x}$ and $\mathbf{A_y}$ produce no moment about **A**.

$$\circlearrowleft + \Sigma M_A = 0;$$

$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

$$N_B = 536.2 \text{ lb} = 536 \text{ lb}$$

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - (536.2 \text{ lb}) \sin 30° = 0$$

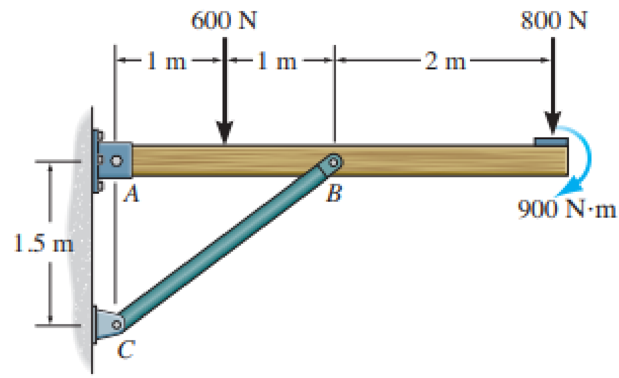
$$A_x = 268 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$$

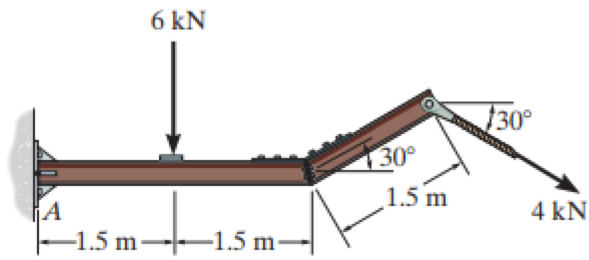
$$A_y = 286 \text{ lb}$$

H.W:

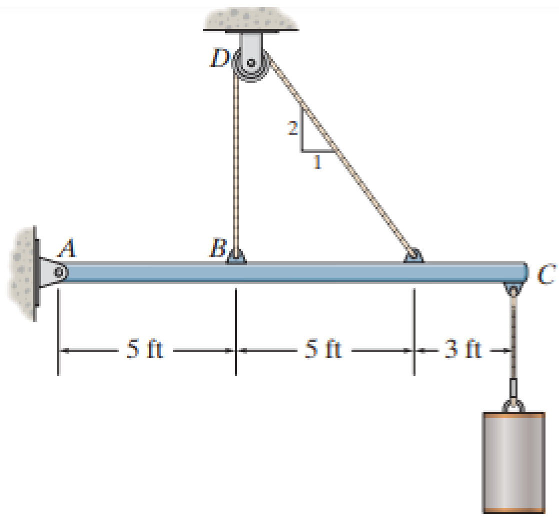
1. The overhanging beam is supported by a pin at **A** and the two-force strut **BC**. Determine the horizontal and vertical components of reaction at **A** and the reaction at **B** on the beam.



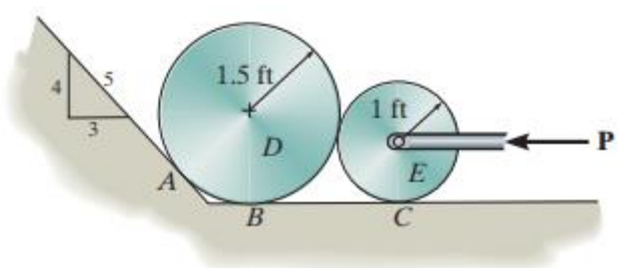
2. Determine the components of the support reactions at the fixed support **A** on the cantilevered beam.



3. Determine the tension in the cable and the horizontal and vertical components of reaction of the pin **A**. The pulley at **D** is frictionless and the cylinder weighs 80 lb.



5. The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. Determine the largest horizontal force P that can be applied to the center of disk E without causing the disk D to move up the incline.



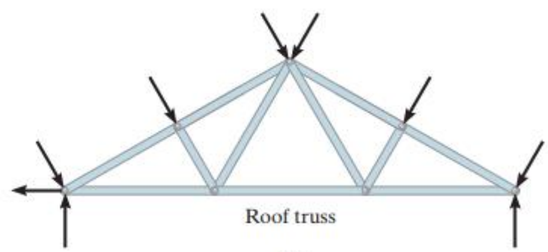
Structural Analysis:

- To show how to determine the forces in the members of a truss using the method of joints and the method of sections.
- To analyze the forces acting on the members of frames and machines composed of pin-connected members.

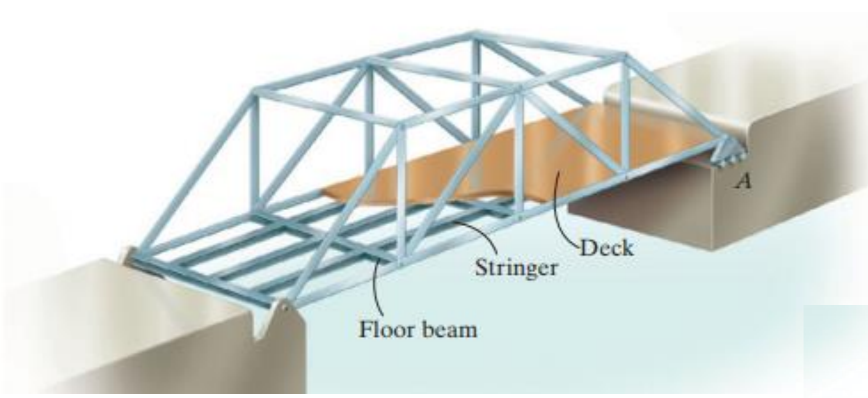
Simple Trusses:

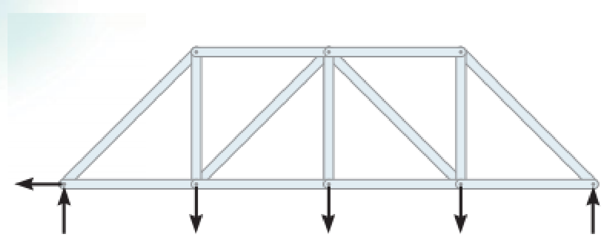
A truss is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. In particular, planar trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in figure a is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss at the joints by means of a series of purlins. Since this loading acts in the same plane as the truss, figure b, the analysis of the forces developed in the truss members will be two-dimensional.

(a)



(b)

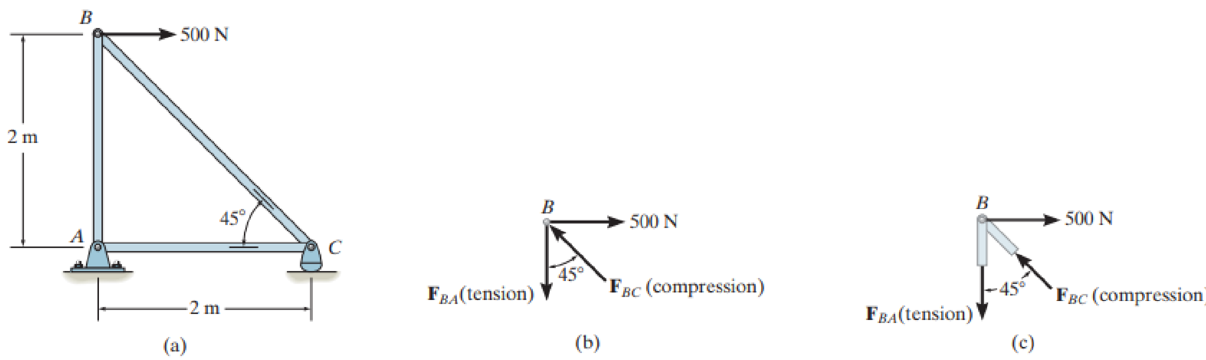



Bridge truss

Assumptions for Design: To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions:

- All loadings are applied at the joints.
- The members are joined together by smooth pins.

The Method of Joints:



Procedure for Analysis: The following procedure provides a means for analyzing a truss using the method of joints.

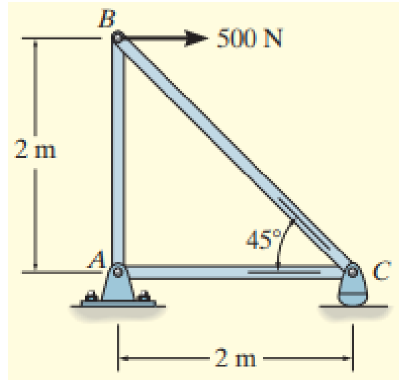
• Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)

• Use one of the two methods described above for establishing the sense of an unknown force.

• Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations Fx = 0 and Fy = 0. Solve for the two unknown member forces and verify their correct sense.

• Using the calculated results, continue to analyze each of the other joints. Remember that a member in compression "pushes" on the joint and a member in tension "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

Example: Determine the force in each member of the truss shown in figure **a** and indicate whether the members are in tension or compression.

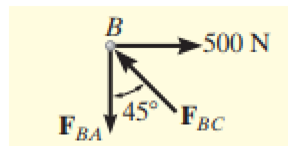


(a)

Solution:

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint B.

Joint B. The free-body diagram of the joint at B is shown in Figure b. Applying the equations of equilibrium, we have:

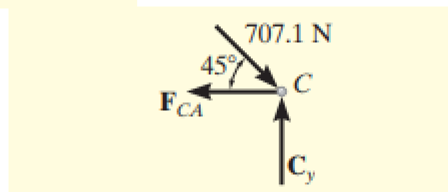


(b)

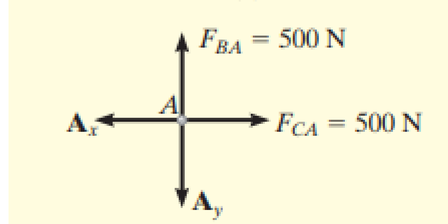
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 500\text{ N} - F_{BC}\sin 45^\circ = 0 \qquad F_{BC} = 707.1\text{ N (C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\cos 45^\circ - F_{BA} = 0 \qquad F_{BA} = 500\text{ N (T)}$$

Joint A. Although it is not necessary, we can determine the components of the support reactions at joint A using the results of FCA and F_{BA}. From the free-body diagram, Figure d, we have



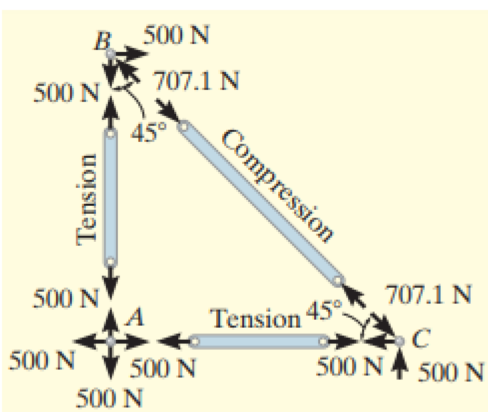
(c)



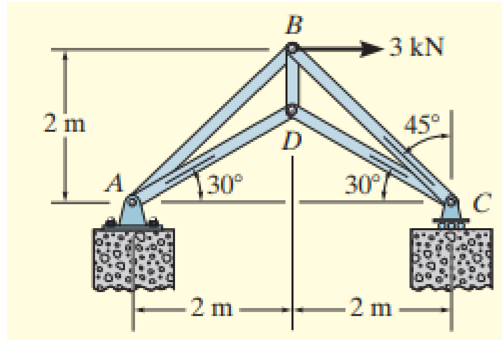
(d)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 500\text{ N} - A_x = 0 \qquad A_x = 500\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 500\text{ N} - A_y = 0 \qquad A_y = 500\text{ N}$$



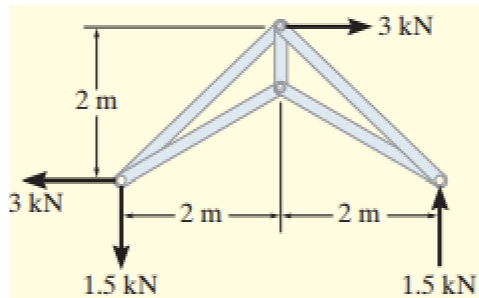
Example: Determine the forces acting in all the members of the truss shown in Figure a.



(a)

Solution:

By inspection, there are more than two unknowns at each joint. Consequently, the support reactions on the truss must first be determined. Show that they have been correctly calculated on the free-body diagram in Figure **b**. We can now begin the analysis at joint **C**. Why?



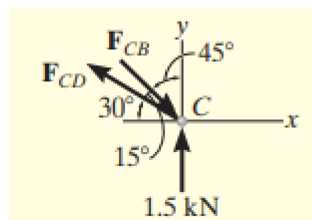
(b)

Joint C. From the free-body diagram, Figure **c**,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_{CD}\cos 30^\circ + F_{CB}\sin 45^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 1.5\text{ kN} + F_{CD}\sin 30^\circ - F_{CB}\cos 45^\circ = 0$$

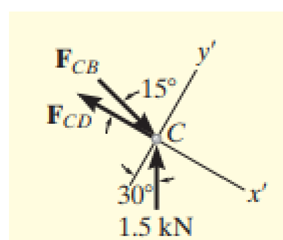
These two equations must be solved simultaneously for each of the two unknowns. Note, however, that a direct solution for one of the unknown forces may be obtained by applying a force summation along an axis that is perpendicular to the direction of the other unknown force. For example, summing forces along the **y′** axis, which is perpendicular to the direction of **F$_{CD}$**, figure d, yields a direct solution for **F$_{CB}$**.



(c)

$$+\nearrow \Sigma F_{y'} = 0; \qquad 1.5\cos 30^\circ\text{ kN} - F_{CB}\sin 15^\circ = 0$$

$$F_{CB} = 5.019\text{ kN} = 5.02\text{ kN (C)}$$



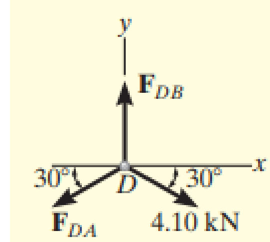
(d)

Then,

$$+\searrow \Sigma F_{x'} = 0;$$

$$-F_{CD} + 5.019\cos 15^\circ - 1.5\sin 30^\circ = 0; \quad F_{CD} = 4.10\text{ kN (T)}$$

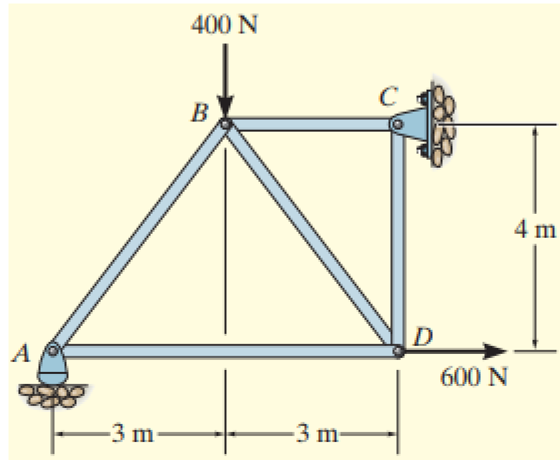
Joint D. We can now proceed to analyze joint D. The free-body diagram is shown in figure **e**.



(e)

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_{DA} \cos 30^\circ + 4.10 \cos 30^\circ \text{ kN} = 0$$
$$F_{DA} = 4.10 \text{ kN} \quad \text{(T)}$$
$$+\uparrow \Sigma F_y = 0; \qquad F_{DB} - 2(4.10 \sin 30^\circ \text{ kN}) = 0$$
$$F_{DB} = 4.10 \text{ kN} \quad \text{(T)}$$

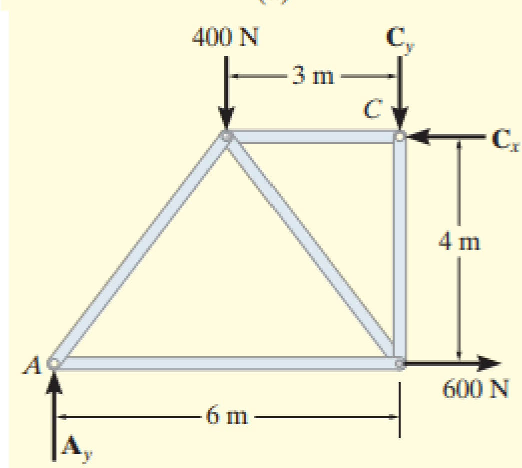
Example: Determine the force in each member of the truss shown in Figure **a**. Indicate whether the members are in tension or compression.



(a)

Solution:

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has at least three unknown forces acting on it. A free-body diagram of the entire truss is given in figure **b**. Applying the equations of equilibrium, we have



(b)

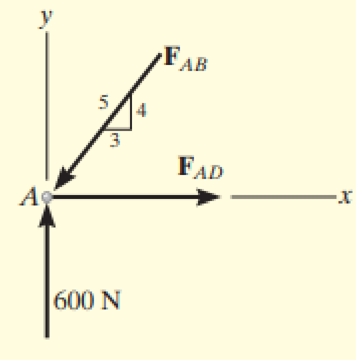
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 600 \text{ N} - C_x = 0 \qquad C_x = 600 \text{ N}$$
$$\circlearrowleft + \Sigma M_C = 0; \qquad -A_y(6 \text{ m}) + 400 \text{ N}(3 \text{ m}) + 600 \text{ N}(4 \text{ m}) = 0$$
$$A_y = 600 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \qquad 600 \text{ N} - 400 \text{ N} - C_y = 0 \qquad C_y = 200 \text{ N}$$

The analysis can now start at either joint **A** or **C**. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints

Joint A. (Figure c). As shown on the free-body diagram, $\mathbf{F}_{AB}$ is assumed to be compressive and $\mathbf{F}_{AD}$ is tensile. Applying the equations of equilibrium, we have:

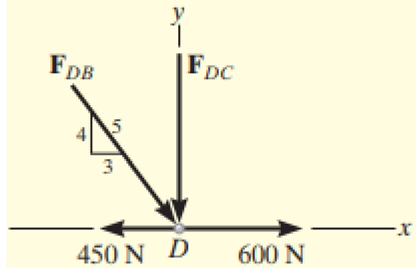
$$+\uparrow \Sigma F_y = 0; \qquad 600\text{ N} - \tfrac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750\text{ N} \quad \text{(C)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AD} - \tfrac{3}{5}(750\text{ N}) = 0 \qquad F_{AD} = 450\text{ N} \quad \text{(T)}$$



(c)

Joint D. (Figure **d**). Using the result for $\mathbf{F}_{AD}$ and summing forces in the horizontal direction, Figure **d**, we have:



(d)

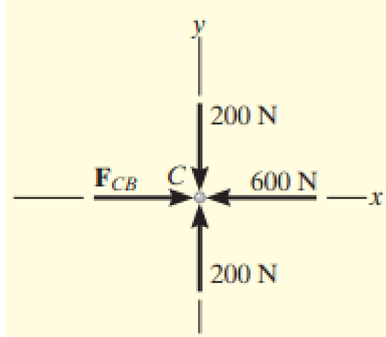
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -450\text{ N} + \tfrac{3}{5}F_{DB} + 600\text{ N} = 0 \qquad F_{DB} = -250\text{ N}$$

$$F_{DB} = 250\text{ N (T)}$$

To determine $\mathbf{F}_{DC}$, we can either correct the sense of $\mathbf{F}_{DB}$ on the free body diagram, and then apply $\Sigma F_y = 0$, or apply this equation and retain the negative sign for $\mathbf{F}_{DB}$, i.e.,

$$+\uparrow \Sigma F_y = 0; \qquad -F_{DC} - \tfrac{4}{5}(-250\text{ N}) = 0 \qquad F_{DC} = 200\text{ N} \quad \text{(C)}$$

Joint C. (Figure **e**).



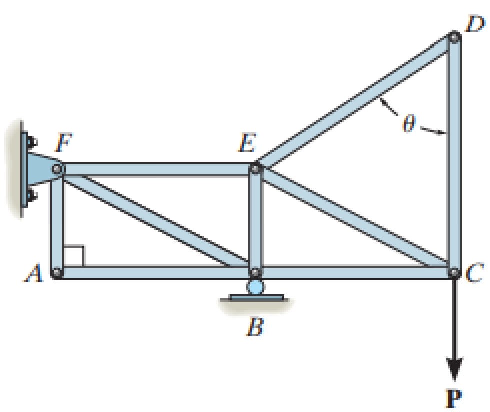
(e)

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CB} - 600\text{ N} = 0 \qquad F_{CB} = 600\text{ N} \quad \text{(C)}$$

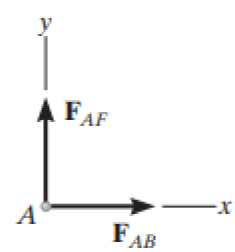
$$+\uparrow \Sigma F_y = 0; \qquad 200\text{ N} - 200\text{ N} \equiv 0 \quad \text{(check)}$$

Zero-Force Members:

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support no loading. These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed. The zero-force members of a truss can generally be found by inspection of each of the joints. For example, consider the truss shown in figure a. If a free-body diagram of the pin at joint A is drawn, figure b, it is seen that members AB and AF are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints F or B simply because there are five unknowns at each of these joints.) In a similar manner, consider the free-body diagram of joint D, figure c. Here again it is seen that DC and DE are zero-force members. From these observations, we can conclude that if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members. The load on the truss in figure a is therefore supported by only five members as shown in figure d

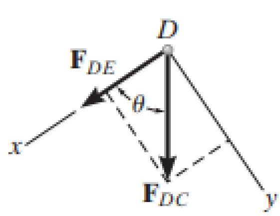


(a)



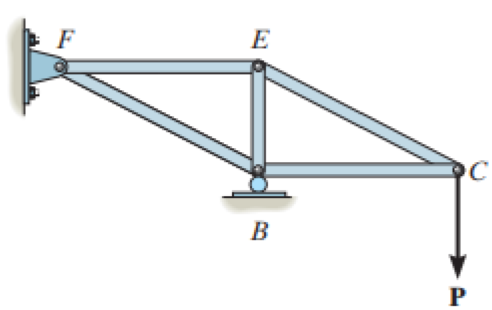
$\overset{+}{\rightarrow} \Sigma F_x = 0;\ F_{AB} = 0$
$+\uparrow \Sigma F_y = 0;\ F_{AF} = 0$

(b)



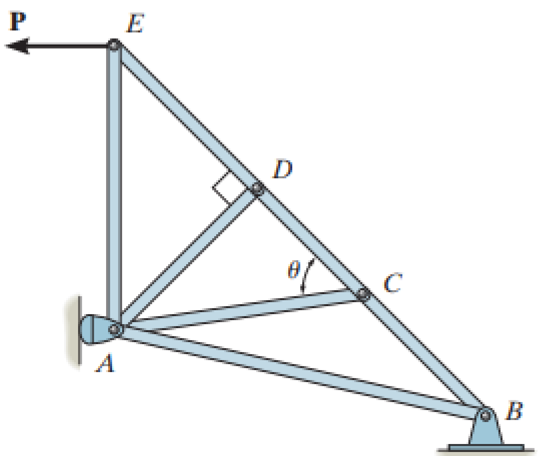
$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\quad F_{DC} = 0 \text{ since } \sin\theta \neq 0$
$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\quad F_{DE} = 0$

(c)

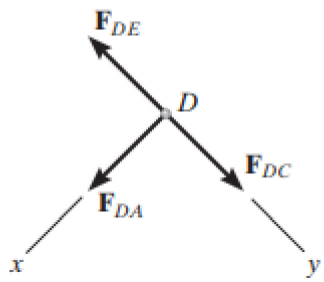


(d)

Now consider the truss shown in figure a. The free-body diagram of the pin at joint D is shown in figure b. By orienting the y axis along members DC and DE and the x axis along member DA, it is seen that DA is a zero-force member. Note that this is also the case for member CA, figure c. In general then, if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint. The truss shown in figure d is therefore suitable for supporting the load **P**.



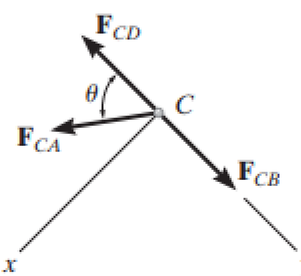
(a)



$+\swarrow \Sigma F_x = 0; \quad F_{DA} = 0$

$+\searrow \Sigma F_y = 0; \quad F_{DC} = F_{DE}$

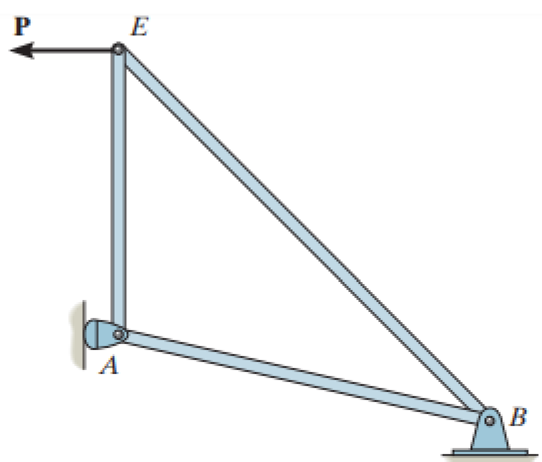
(b)



$+\swarrow \Sigma F_x = 0; \quad F_{CA} \sin\theta = 0; \quad F_{CA} = 0 \text{ since } \sin\theta \neq 0;$

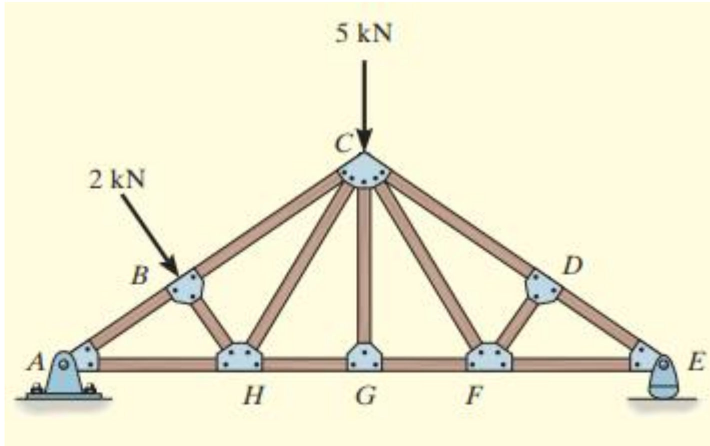
$+\searrow \Sigma F_y = 0; \quad F_{CB} = F_{CD}$

(c)



(d)

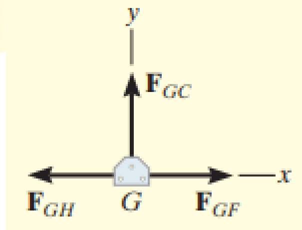
Example: Using the method of joints, determine all the zero-force members of the Fink roof truss shown in figure a. Assume all joints are pin connected.



(a)

Solution:

Look for joint geometries that have three members for which two are collinear. We have Joint G. (Figure b).

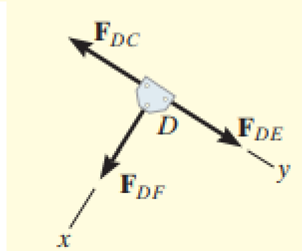


(b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{GC} = 0$$

Realize that we could not conclude that GC is a zero-force member by considering joint C, where there are five unknowns. The fact that GC is a zero-force member means that the 5-kN load at C must be supported by members CB, CH , CF , and CD.

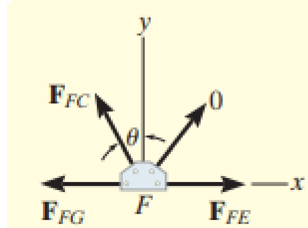
Joint D. (Figure c)



(c)

$$+\swarrow \Sigma F_x = 0; \qquad F_{DF} = 0$$

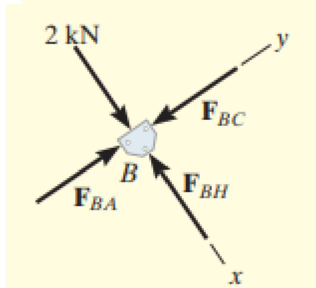
Joint F. (Figure d).:



(d)

$$+\uparrow \Sigma F_y = 0; \qquad F_{FC}\cos\theta = 0 \quad \text{Since } \theta \neq 90^\circ, \quad F_{FC} = 0$$

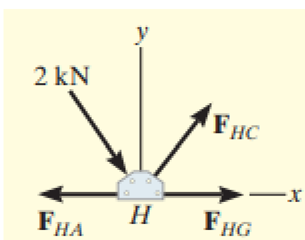
NOTE: If joint B is analyzed, Figure e,



(e)

$$+\searrow \Sigma F_x = 0; \qquad 2\text{ kN} - F_{BH} = 0 \quad F_{BH} = 2\text{ kN} \quad (C)$$

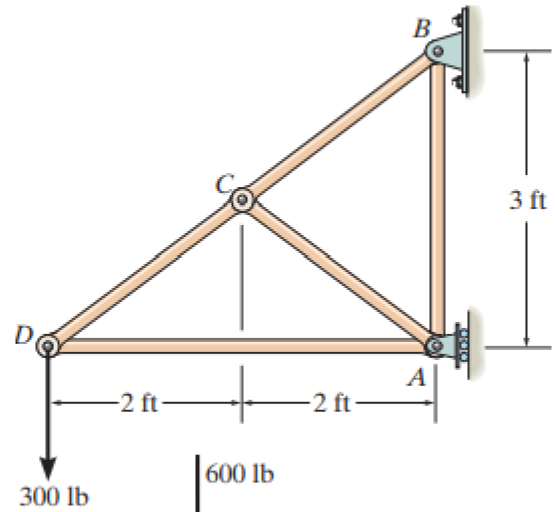
Also, **F_{HC}** must satisfy ΣFy = 0, figure f, and therefore HC is not a zero-force member



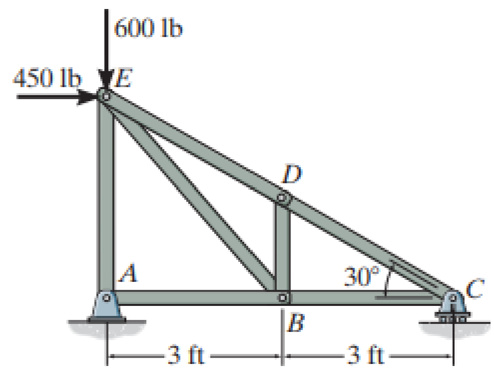
(f)

H.W:

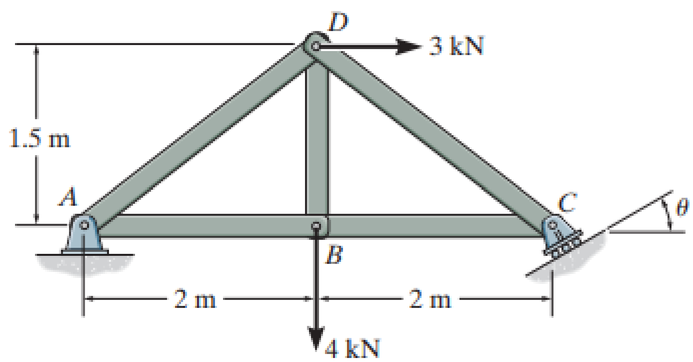
1. Determine the force in each member of the truss. State if the members are in tension or compression



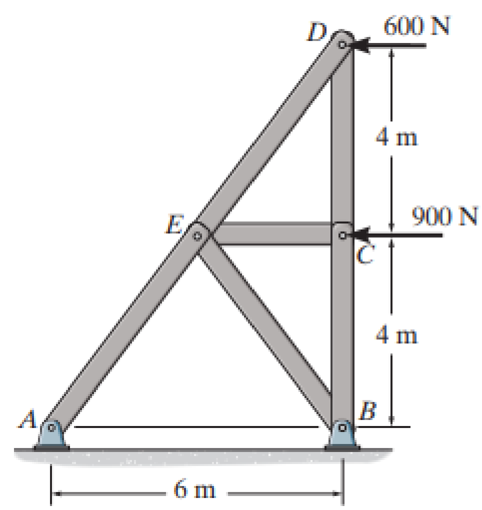
2. Determine the force in each member of the truss. State if the members are in tension or compression.



3. Determine the force in each member of the truss, and state if the members are in tension or compression. Set θ = 30.

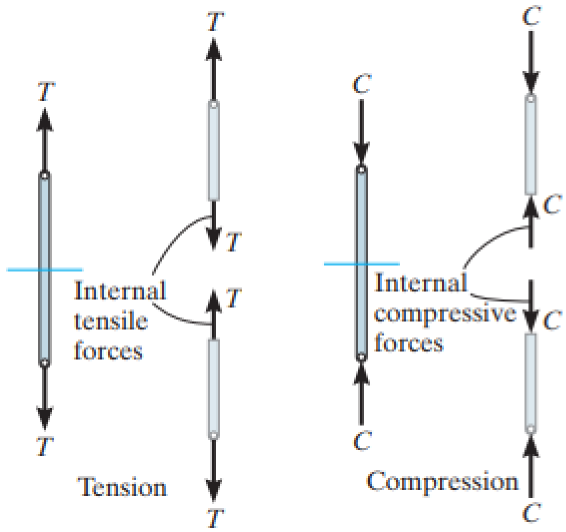


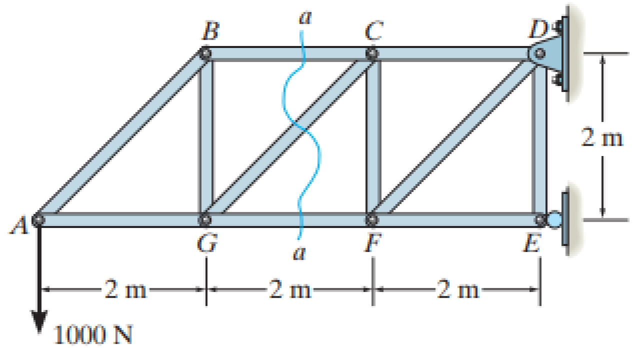
4. Determine the force in each member of the truss, and state if the members are in tension or compression



Method of Section:

When we need to find the force in only a few members of a truss, we can analyze the truss using the method of sections. It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. For example, consider the two truss members shown on the left in figure. If the forces within the members are to be determined, then an imaginary section, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension (**T**) be subjected to a "pull," whereas the member in compression (**C**) is subjected to a "push." The method of sections can also be used to "cut" or section the members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the equations of equilibrium to that part to determine the member forces at the "cut section." Since only three independent equilibrium equations ($\boldsymbol{\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0}$) can be applied to the free-body diagram of any segment, then we should try to select a section that, in general, passes through not more than three members in which the forces are unknown. For example, consider the truss in figure a . If the forces in members **BC**, **GC**, and **GF** are to be determined, then section **a-a** would be appropriate. The free-body diagrams of the two segments are shown in Figures b and c . Note that the line of action of each member force is specified from the geometry of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton's third law. Members BC and GC are assumed to be in tension since they are subjected to a "pull," whereas GF in compression since it is subjected to a "push." The three unknown member forces F_{BC}, F_{GC}, and F_{GF} can be obtained by applying the three equilibrium equations to the free-body diagram in Figure b. If, however, the free-body diagram in Figure c is considered, the three support reactions D_x, D_y and E_x will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the entire truss.

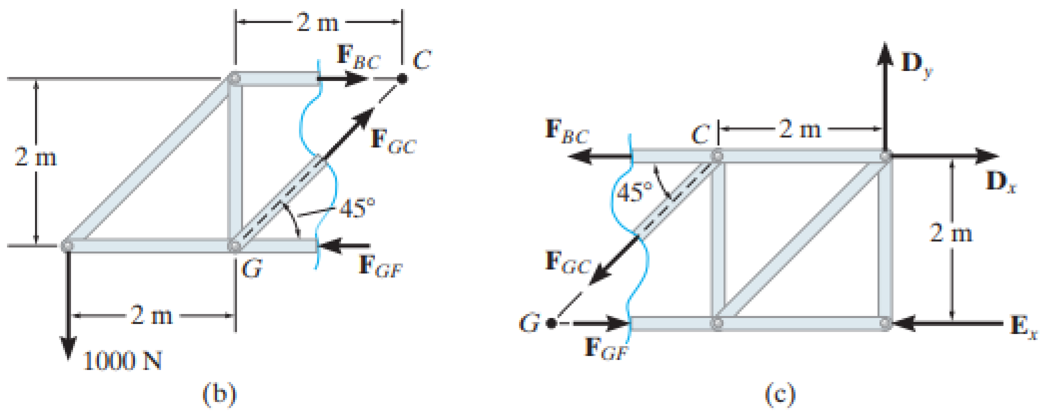




When applying the equilibrium equations, we should carefully consider ways of writing the equations so as to yield a direct solution for each of the unknowns, rather than having to solve simultaneous equations. For example, using the truss segment in figure b and summing moments about C would yield a direct solution for $\mathbf{F}_{GF}$ since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GC}$ create zero moment about C. Likewise, $\mathbf{F}_{BC}$ can be directly obtained by summing moments about G. Finally, $\mathbf{F}_{GC}$ can be found directly from a force summation in the vertical direction since $\mathbf{F}_{GF}$ and $\mathbf{F}_{BC}$ have no vertical components. This ability to determine directly the force in a particular truss member is one of the main advantages of using the method of sections. * As in the method of joints, there are two ways in which we can determine the correct sense of an unknown member force:

The correct sense of an unknown member force can in many cases be determined “by inspection.” For example, $\mathbf{F}_{BC}$ is a tensile force as represented in figure b since moment equilibrium about G requires that $\mathbf{F}_{BC}$ create a moment opposite to that of the 1000-N force. Also, $\mathbf{F}_{GC}$ is tensile since its vertical component must balance the 1000-N force which acts downward. In more complicated cases, the sense of an unknown member force may be assumed. If the solution yields a negative scalar, it indicates that the force’s sense is opposite to that shown on the free-body diagram.

• Always assume that the unknown member forces at the cut section are tensile forces, i.e., "pulling" on the member. By doing this, the numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression.



Procedure for Analysis: The forces in the members of a truss may be determined by the method of sections using the following procedure:

Free-Body Diagram. • Make a decision on how to "cut" or section the truss through the members where forces are to be determined.

• Before isolating the appropriate section, it may first be necessary to determine the truss's support reactions. If this is done then the three equilibrium equations will be available to solve for member forces at the section.

• Draw the free-body diagram of that segment of the sectioned truss which has the least number of forces acting on it.

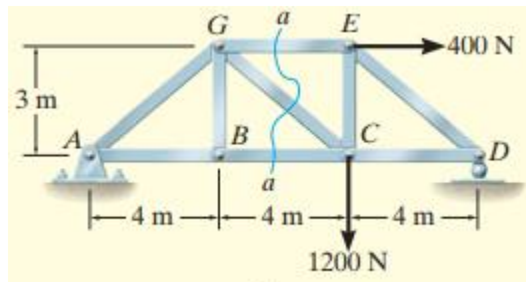
• Use one of the two methods described above for establishing the sense of the unknown member forces.

Equations of Equilibrium.

• Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces, so that the third unknown force can be determined directly from the moment equation.

• If two of the unknown forces are parallel, forces may be summed perpendicular to the direction of these unknowns to determine directly the third unknown force.

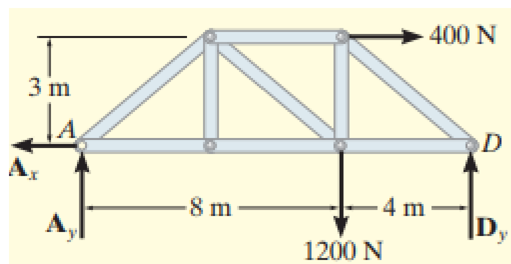
Example: Determine the force in members GE, GC, and BC of the truss shown in figure Indicate whether the members are in tension or compression.



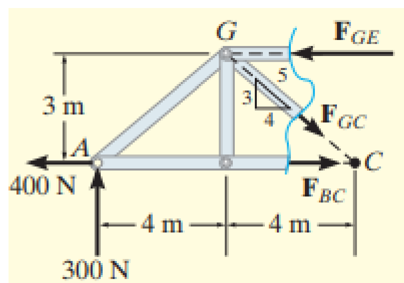
(a)

Solution:

Section a-a in figure a has been chosen since it cuts through the three members whose forces are to be determined. In order to use the method of sections, however, it is first necessary to determine the external reactions at A or D. Why? A free-body diagram of the entire truss is shown in figure b. Applying the equations of equilibrium, we have:



(b)



(c)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 400\text{ N} - A_x = 0 \qquad A_x = 400\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -1200\text{ N}(8\text{ m}) - 400\text{ N}(3\text{ m}) + D_y(12\text{ m}) = 0$$

$$D_y = 900\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 1200\text{ N} + 900\text{ N} = 0 \qquad A_y = 300\text{ N}$$

Free-Body Diagram: For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, figure c .

Equations of Equilibrium. Summing moments about point G eliminates FGE and FGC and yields a direct solution for $\mathbf{F_{BC}}$

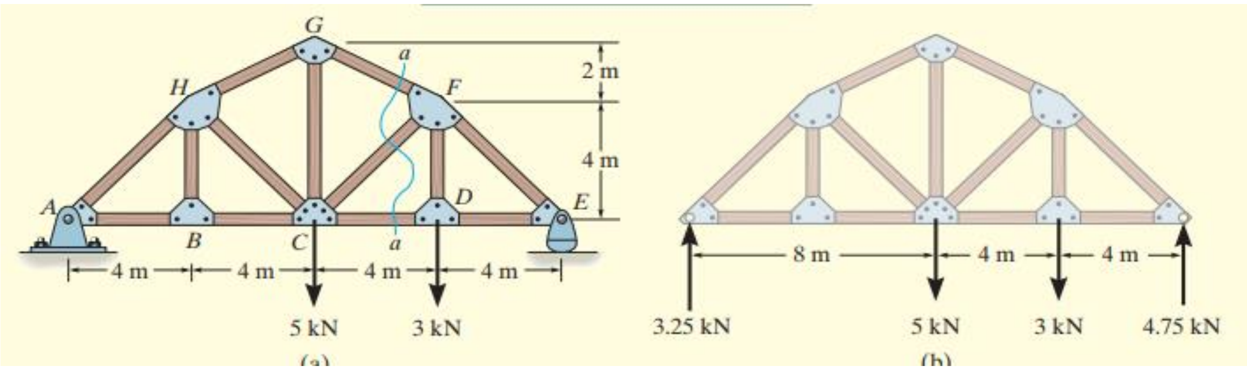
$$\circlearrowleft +\Sigma M_G = 0; \quad -300\text{ N}(4\text{ m}) - 400\text{ N}(3\text{ m}) + F_{BC}(3\text{ m}) = 0$$
$$F_{BC} = 800\text{ N} \quad \text{(T)}$$

In the same manner, by summing moments about point C we obtain a direct solution for $\mathbf{F_{GE}}$.

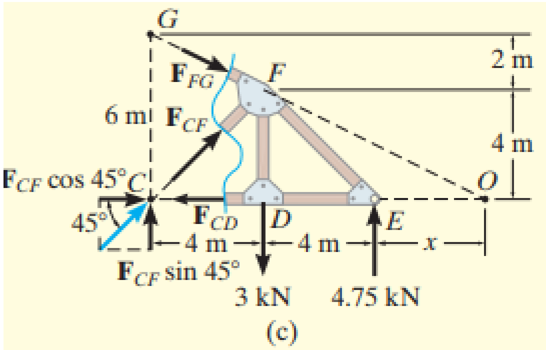
$$\circlearrowleft +\Sigma M_C = 0; \quad -300\text{ N}(8\text{ m}) + F_{GE}(3\text{ m}) = 0$$
$$F_{GE} = 800\text{ N} \quad \text{(C)}$$

Example: Determine the force in member CF of the truss shown in figure a. Indicate whether the member is in tension or compression. Assume each member is pin connected.

Solution:



Free-Body Diagram. Section aa in figure a will be used since this section will “expose” the internal force in member CF as “external” on the free-body diagram of either the right or left portion of the truss. It is first necessary, however, to determine the support reactions on either the left or right side. Verify the results shown on the free-

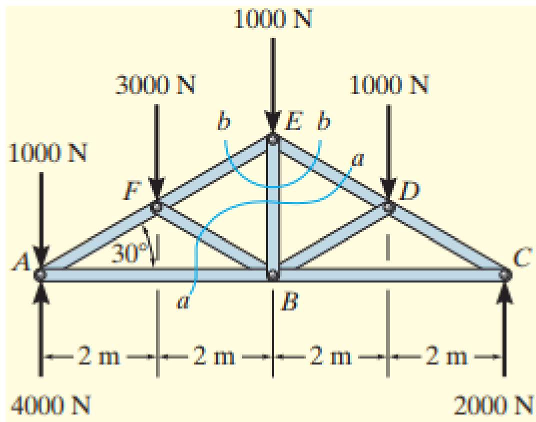


body diagram in figure b. The free-body diagram of the right portion of the truss, which is the easiest to analyze, is shown in figure c. There are three unknowns, $\mathbf{F_{FG}}$, $\mathbf{F_{CF}}$, and $\mathbf{F_{CD}}$. Equations of Equilibrium. We will apply the moment equation about point O in order to eliminate the two unknowns $\mathbf{F_{FG}}$ and $\mathbf{F_{CD}}$. The location of point O measured from E can be determined from proportional triangles, i.e., 4/(4 + x) = 6/(8 + x), x = 4 m. Or, stated in another manner, the slope of member GF has a drop of 2 m to a horizontal distance of 4 m. Since $\mathbf{F_D}$ is 4 m, figure c, then from **D** to **O** the distance must be 8 m. An easy way to determine the moment of FCF about point O is to use the principle of transmissibility and slide $\mathbf{F_{CF}}$ to point C, and then resolve $\mathbf{F_{CF}}$ into its two rectangular components. We have:

$$\circlearrowleft + \Sigma M_O = 0;$$
$$-F_{CF} \sin 45°(12 \text{ m}) + (3 \text{ kN})(8 \text{ m}) - (4.75 \text{ kN})(4 \text{ m}) = 0$$

$$F_{CF} = 0.589 \text{ kN} \quad \text{(C)}$$

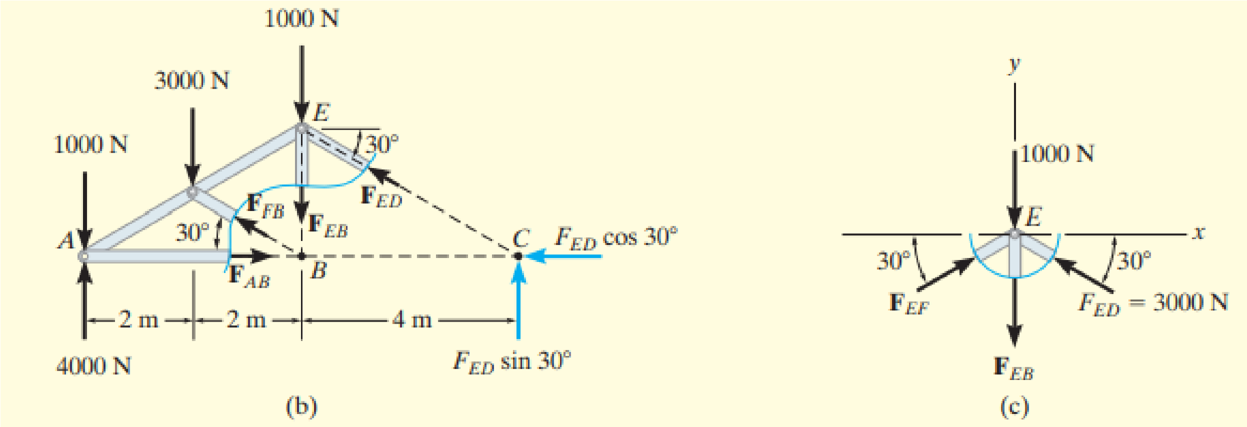
Example: Determine the force in member EB of the roof truss shown in figure a. Indicate whether the member is in tension or compression.



(a)

Solution:

By the method of sections, any imaginary section that cuts through EB, figure a, will also have to cut through three other members for which the forces are unknown. For example, section a-a cuts through ED, EB, FB, and AB. If a free-body diagram of the left side of this section is considered, figure b , it is possible to obtain $\mathbf{F_{ED}}$ by summing moments about B to eliminate the other three unknowns; however, $\mathbf{F_{EB}}$ cannot be determined from the remaining two equilibrium equations. One possible way of obtaining $\mathbf{F_{EB}}$ is first to determine $\mathbf{F_{ED}}$ from section a-a, then use this result on section b-b, figure a, which is shown in figure c. Here the force system is concurrent and our sectioned free-body diagram is the same as the free-body diagram for the joint at E.



Equations of Equilibrium. In order to determine the moment of $\mathbf{F_{ED}}$ about point B, figure b , we will use the principle of transmissibility and slide the force to point C and then resolve it into its rectangular components as shown. Therefore,

$$\circlearrowleft +\Sigma M_B = 0; \qquad 1000\text{ N}(4\text{ m}) + 3000\text{ N}(2\text{ m}) - 4000\text{ N}(4\text{ m}) + F_{ED}\sin 30°(4\text{ m}) = 0$$

$$F_{ED} = 3000\text{ N} \quad (\text{C})$$

Considering now the free-body diagram of section *bb*, Fig. 6–18*c*, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{EF}\cos 30° - 3000\cos 30°\text{ N} = 0$$

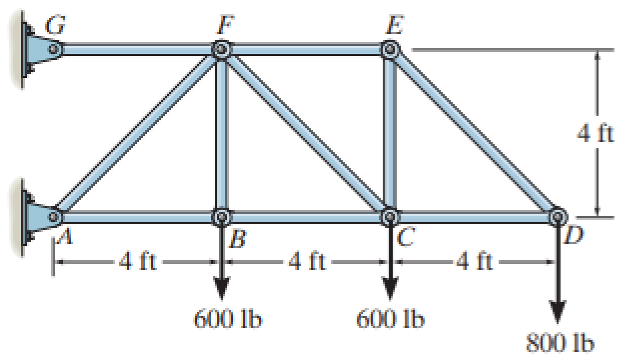
$$F_{EF} = 3000\text{ N} \quad (\text{C})$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(3000\sin 30°\text{ N}) - 1000\text{ N} - F_{EB} = 0$$

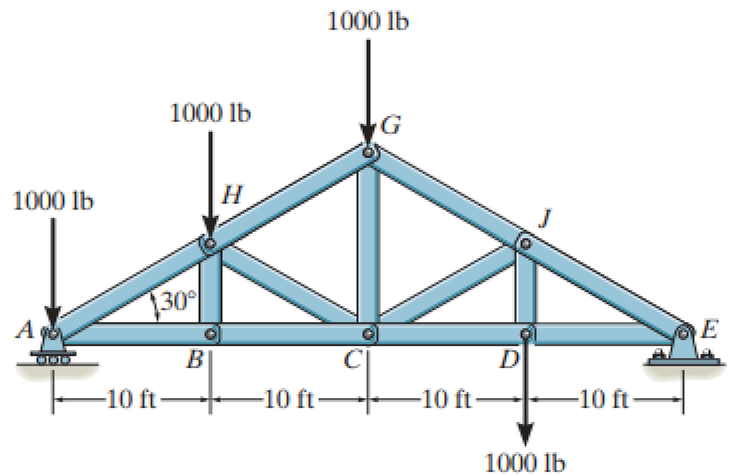
$$F_{EB} = 2000\text{ N} \quad (\text{T})$$

H.W:

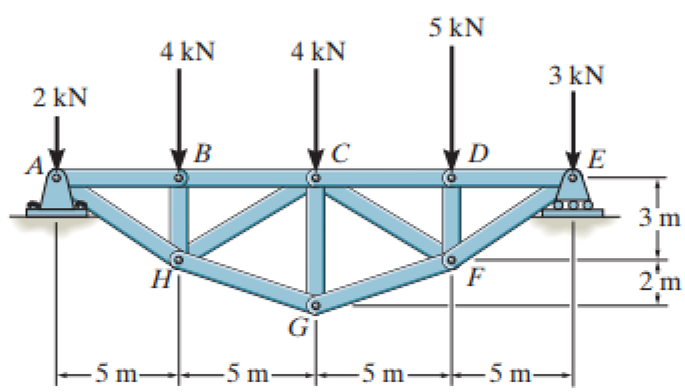
1. Determine the force in members BC, CF, and FE. State if the members are in tension or compression.



2. Determine the force in member GC of the truss and state if this member is in tension or compression.



3. Determine the force in members CD, CF, and CG and state if these members are in tension or compression.



Frames and Machines:

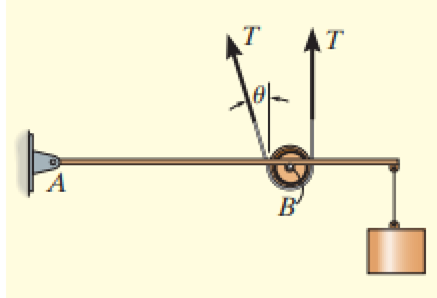
Frames and machines are two types of structures which are often composed of pin-connected multi-force members, i.e., members that are subjected to more than two forces. Frames are used to support loads, whereas machines contain moving parts and are designed to transmit and alter the effect of forces. Provided a frame or machine contains no more supports or members than are necessary to prevent its collapse, the forces acting at the joints and supports can be determined by applying the equations of equilibrium to each of its members. Once these forces are obtained, it is then possible to design the size of the members, connections, and supports using the theory of mechanics of materials and an appropriate engineering design code.

Free-Body Diagrams: In order to determine the forces acting at the joints and supports of a frame or machine, the structure must be disassembled and the free-body diagrams of its parts must be drawn. The following important points must be observed: • Isolate each part by drawing its outlined shape. Then show all the forces and/or couple moments that act on the part. Make sure to label or identify each known and unknown force and couple moment with reference to an established x, y coordinate system. Also, indicate any dimensions used for taking moments. Most often the equations of equilibrium are easier to apply if the forces are represented by their rectangular components. As usual, the sense of an unknown force or couple moment can be assumed. • Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. By recognizing the two-force members, we can avoid solving an unnecessary number of equilibrium equations. • Forces common to any two contacting members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a "system" of connected members, then these forces are "internal" and are not shown on the free-body diagram of the system; however, if the free-body diagram of each member is drawn, the forces are "external" and must be shown as equal in magnitude and opposite in direction on each of the two free-body diagrams. The following examples graphically illustrate how to draw the free-body diagrams of a dismembered frame or machine. In all cases, the weight of the members is neglected.

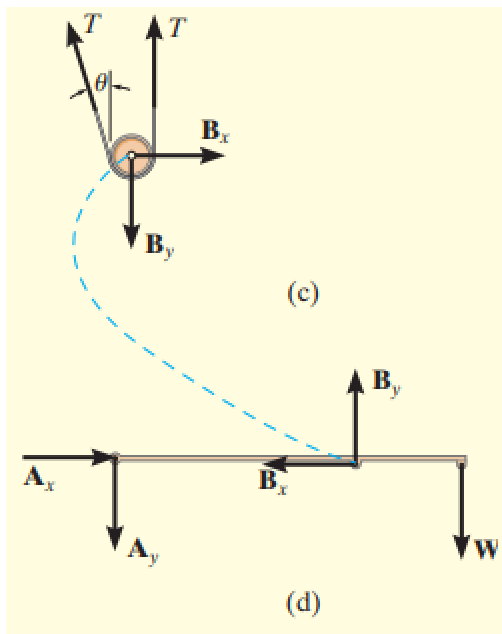
Example: A constant tension in the conveyor belt is maintained by using the device shown in figure a. Draw the free-body diagrams of the frame and the cylinder (or pulley) that the belt surrounds. The suspended block has a weight of W.

(a)

Solution: The idealized model of the device is shown in figure b . Here the angle θ is assumed to be known. From this model, the free-body diagrams of the pulley and frame are shown in figures c and d, respectively. Note that the force components $\mathbf{B_x}$ and B_y that the pin at B exerts on the pulley must be equal but opposite to the ones acting on the frame.



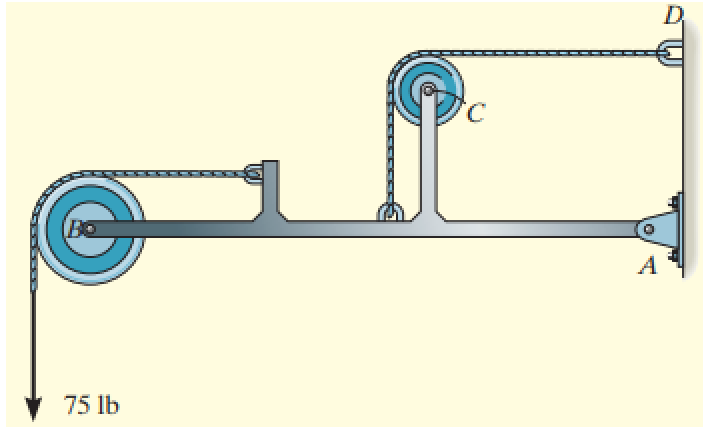
(b)



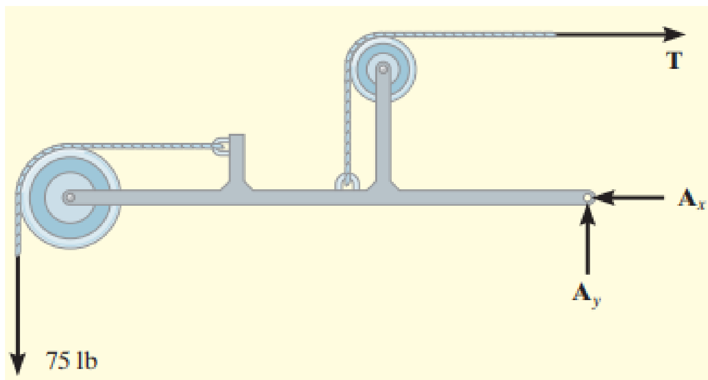
(c)

(d)

Example: For the frame shown in figure a , draw the free-body diagrams of (a) the entire frame including the pulleys and cords, (b) the frame without the pulleys and cords, and (c) each of the pulleys.



(a)



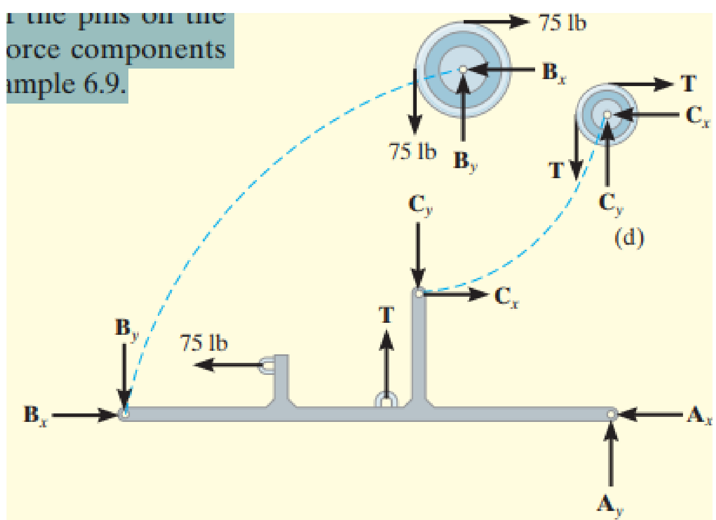
(b)

Solution:

Part (a). When the entire frame including the pulleys and cords is considered, the interactions at the points where the pulleys and cords are connected to the frame become pairs of internal forces which cancel each other and therefore are not shown on the free-body diagram, figure b.

Part (b). When the cords and pulleys are removed, their effect on the frame must be shown, figure c.

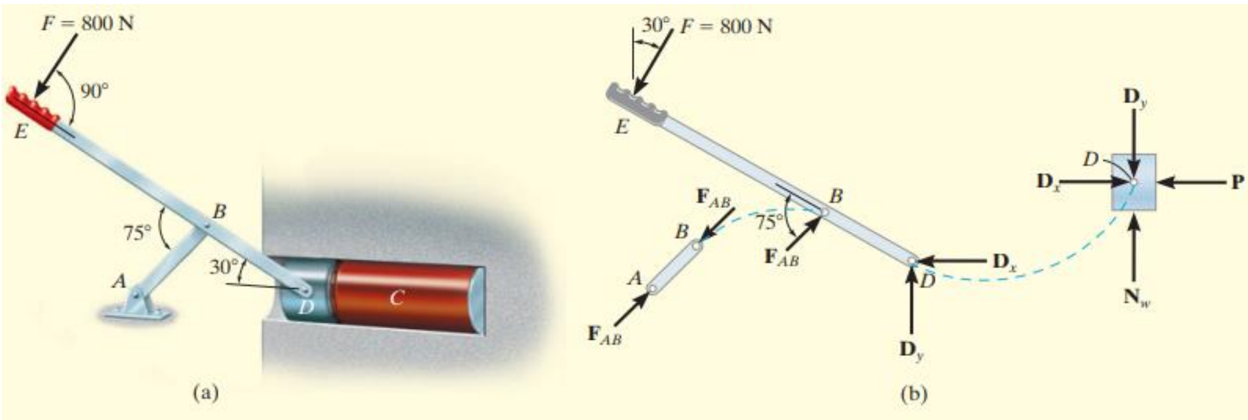
Part (c). The force components B_x, B_y, C_x, C_y of the pins on the pulleys, figure d, are equal but opposite to the force components exerted by the pins on the frame, figure c.



(c)

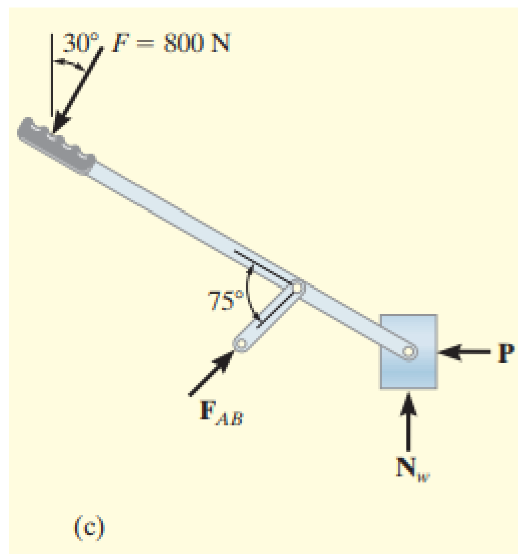
Example: Draw the free-body diagram of each part of the smooth piston and link mechanism used to crush recycled cans, figure a.

Solution:



(a) (b)

By inspection, member AB is a two-force member. The free-body diagrams of the three parts are shown in figure b. Since the pins at B and D connect only two parts together, the forces there are shown as equal but opposite on the separate free-body diagrams of their connected members. In particular, four components of force act on the piston: **D_x** and **D_y** represent the effect of the pin (or lever **E_{BD}**), **N_w** is the resultant force of the wall support, and P is the resultant compressive force caused by the can C. The directional sense of each of the unknown forces is assumed, and the correct sense will be established after the equations of equilibrium are applied.



(c)

Procedure for Analysis:

The joint reactions on frames or machines (structures) composed of multi-force members can be determined using the following procedure.

Free-Body Diagram.

• Draw the free-body diagram of the entire frame or machine, a portion of it, or each of its members. The choice should be made so that it leads to the most direct solution of the problem.

• When the free-body diagram of a group of members of a frame or machine is drawn, the forces between the connected parts of this group are internal forces and are not shown on the free-body diagram of the group.

• Forces common to two members which are in contact act with equal magnitude but opposite sense on the respective free-body diagrams of the members.

• Two-force members, regardless of their shape, have equal but opposite collinear forces acting at the ends of the member.

• In many cases it is possible to tell by inspection the proper sense of the unknown forces acting on a member; however, if this seems difficult, the sense can be assumed.

• Remember that a couple moment is a free vector and can act at any point on the free-body diagram. Also, a force is a sliding vector and can act at any point along its line of action.

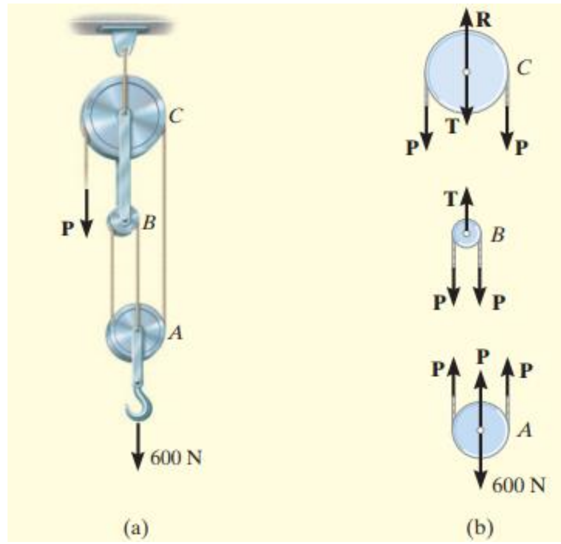
Equations of Equilibrium.

• Count the number of unknowns and compare it to the total number of equilibrium equations that are available. In two dimensions, there are three equilibrium equations that can be written for each member.

• Sum moments about a point that lies at the intersection of the lines of action of as many of the unknown forces as possible.

• If the solution of a force or couple moment magnitude is found to be negative, it means the sense of the force is the reverse of that shown on the free-body diagram.

Example: Determine the tension in the cables and also the force **P** required to support the 600-N force using the frictionless pulley system shown in figure a.



Solution:

Free-Body Diagram.

A free-body diagram of each pulley including its pin and a portion of the contacting cable is shown in figure b. Since the cable is continuous, it has a constant tension P acting throughout its length. The link connection between pulleys B and C is a two-force member, and therefore it has an unknown tension T acting on it. Notice that the principle of action, equal but opposite reaction must be carefully observed for forces P and T when the separate free body diagrams are drawn. Equations of Equilibrium. The three unknowns are obtained as follows:

Pulley A

$+\uparrow \Sigma F_y = 0; \qquad 3P - 600\text{ N} = 0 \qquad P = 200\text{ N}$

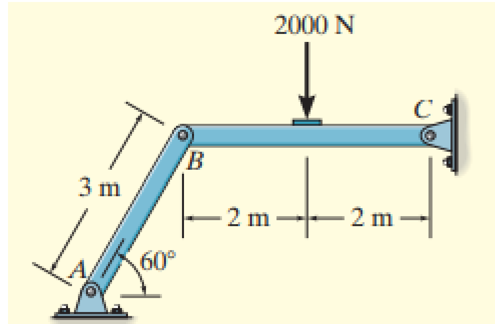
Pulley B

$+\uparrow \Sigma F_y = 0; \qquad T - 2P = 0 \qquad T = 400\text{ N}$

Pulley C

$+\uparrow \Sigma F_y = 0; \qquad R - 2P - T = 0 \qquad R = 800\text{ N}$

Example: Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in figure a.



(a)

Solution:

Free-Body Diagrams. By inspection it can be seen that AB is a two force member. The free-body diagrams are shown in figure b.

Equations of Equilibrium. The three unknowns can be determined by applying the three equations of equilibrium to member CB

$$\circlearrowleft +\Sigma M_C = 0;\ \ 2000\text{ N}(2\text{ m}) - (F_{AB}\sin 60°)(4\text{ m}) = 0 \quad F_{AB} = 1154.7\text{ N}$$

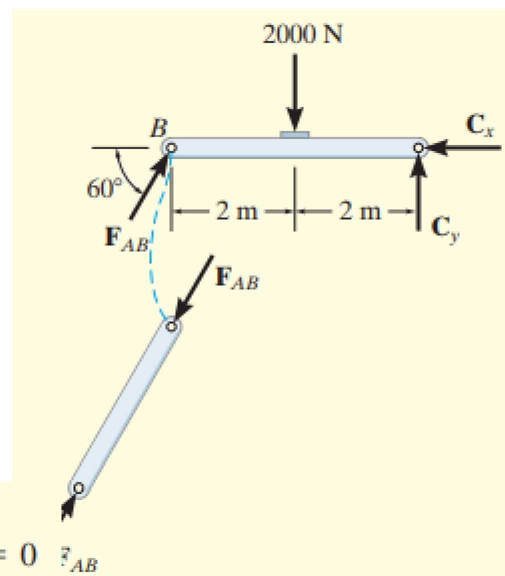
$$\overset{+}{\rightarrow}\Sigma F_x = 0;\ \ 1154.7\cos 60°\text{ N} - C_x = 0 \quad C_x = 577\text{ N}$$

$$+\uparrow\Sigma F_y = 0;\ \ 1154.7\sin 60°\text{ N} - 2000\text{ N} + C_y = 0$$

$$C_y = 1000\text{ N}$$

SOLUTION II:

Free-Body Diagrams. If one does not recognize that AB is a two force member, then more work is involved in solving this problem. The free-body diagrams are shown in figure c . Equations of Equilibrium. The six unknowns are determined by applying the three equations of equilibrium to each member.



(b)

Member AB

$$\circlearrowleft +\Sigma M_A = 0;\quad B_x(3\sin 60°\text{ m}) - B_y(3\cos 60°\text{ m}) = 0$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad A_x - B_x = 0$$

$$+\uparrow\Sigma F_y = 0;\quad A_y - B_y = 0$$

Member BC

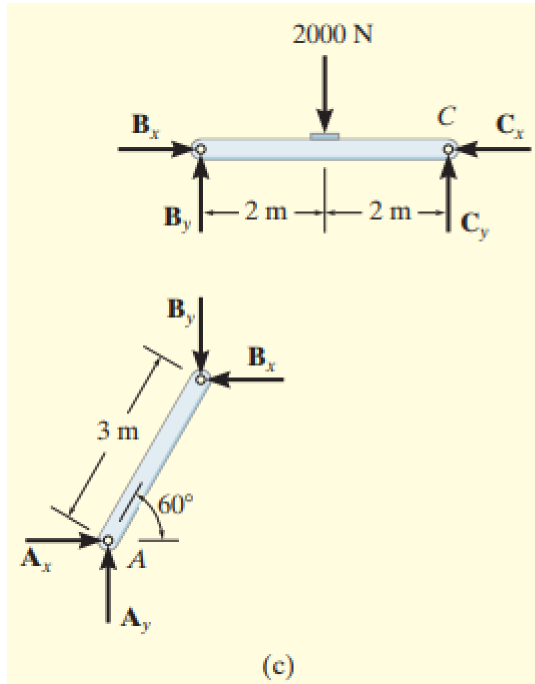
$$\circlearrowleft +\Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - B_y(4\text{ m}) = 0$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad B_x - C_x = 0$$

$$+\uparrow\Sigma F_y = 0;\quad B_y - 2000\text{ N} + C_y = 0$$

The results for C_x and C_y can be determined by solving these equations in the following sequence: 4, 1, 5, then 6. The results are

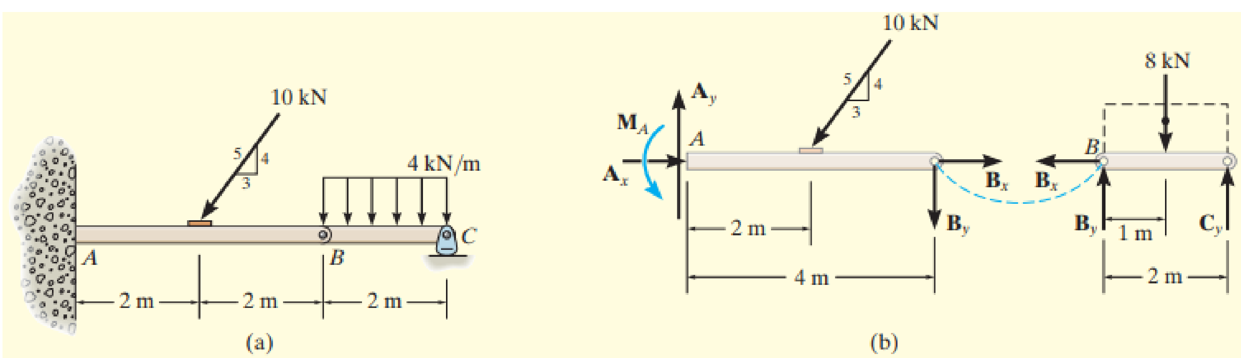
$$B_y = 1000 \text{ N}$$
$$B_x = 577 \text{ N}$$
$$C_x = 577 \text{ N}$$
$$C_y = 1000 \text{ N}$$



(c)

Example: The compound beam shown in figure a, is pin connected at B. Determine the components of reaction at its supports. Neglect its weight and thickness.

Solution:



(a) (b)

Free-Body Diagrams: By inspection, if we consider a free-body diagram of the entire beam ABC, there will be three unknown reactions at A and one at C. These four unknowns cannot all be obtained from the three available equations of equilibrium, and so for the solution it will become necessary to dismember the beam into its two segments, as shown in figure b.

Equations of Equilibrium. The six unknowns are determined as follows:

Segment BC

$$\xleftarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -8\text{ kN}(1\text{ m}) + C_y(2\text{ m}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 8\text{ kN} + C_y = 0$$

Segment AB

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - (10\text{ kN})\left(\tfrac{3}{5}\right) + B_x = 0$$

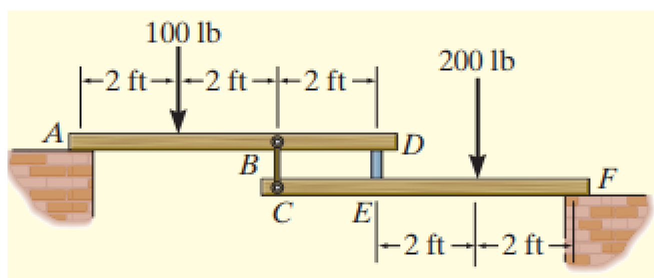
$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - (10\text{ kN})\left(\tfrac{4}{5}\right)(2\text{ m}) - B_y(4\text{ m}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - (10\text{ kN})\left(\tfrac{4}{5}\right) - B_y = 0$$

Solving each of these equations successively, using previously calculated results, we obtain:

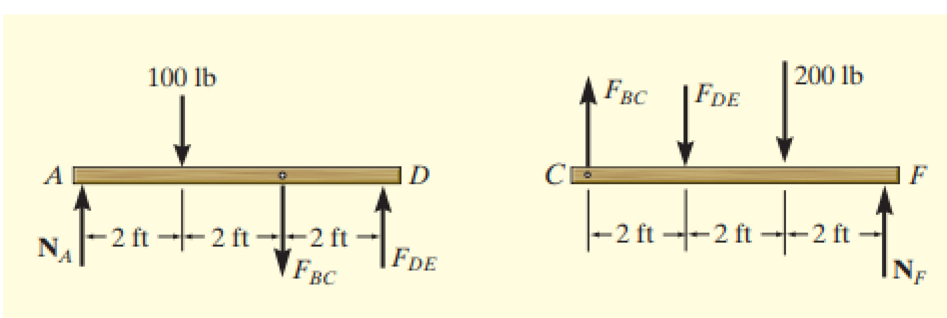
$$A_x = 6\text{ kN} \qquad A_y = 12\text{ kN} \qquad M_A = 32\text{ kN}\cdot\text{m}$$
$$B_x = 0 \qquad B_y = 4\text{ kN}$$
$$C_y = 4\text{ kN}$$

Example: The two planks in figure a are connected together by cable BC and a smooth spacer DE. Determine the reactions at the smooth supports A and F, and also find the force developed in the cable and spacer



(a)

Solution:



(b)

Free-Body Diagrams. The free-body diagram of each plank is shown in Figure b. It is important to apply Newton's third law to the interaction forces $\mathbf{F_{BC}}$ and $\mathbf{F_{DE}}$ as shown. Equations of Equilibrium. For plank AD,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{DE}(6\text{ ft}) - F_{BC}(4\text{ ft}) - 100\text{ lb}\,(2\text{ ft}) = 0$$

For plank CF,

$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{DE}(4\text{ ft}) - F_{BC}(6\text{ ft}) + 200\text{ lb}\,(2\text{ ft}) = 0$$

Solving simultaneously,

$$F_{DE} = 140\text{ lb} \quad F_{BC} = 160\text{ lb}$$

Using these results, for plank AD,

$$+\uparrow \Sigma F_y = 0; \qquad N_A + 140\text{ lb} - 160\text{ lb} - 100\text{ lb} = 0$$

$$N_A = 120\text{ lb}$$

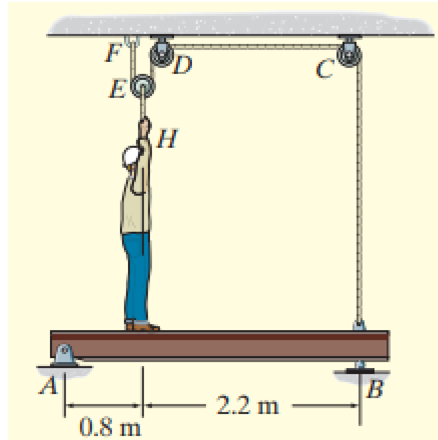
And for plank CF,

$$+\uparrow \Sigma F_y = 0; \qquad N_F + 160\text{ lb} - 140\text{ lb} - 200\text{ lb} = 0$$

$$N_F = 180\text{ lb}$$

NOTE: Draw the free-body diagram of the system of both planks and apply ΣMA = 0 to determine NF. Then use the free-body diagram of $\mathbf{C_{EF}}$ to determine $\mathbf{F_{DE}}$ and $\mathbf{F_{BC}}$.

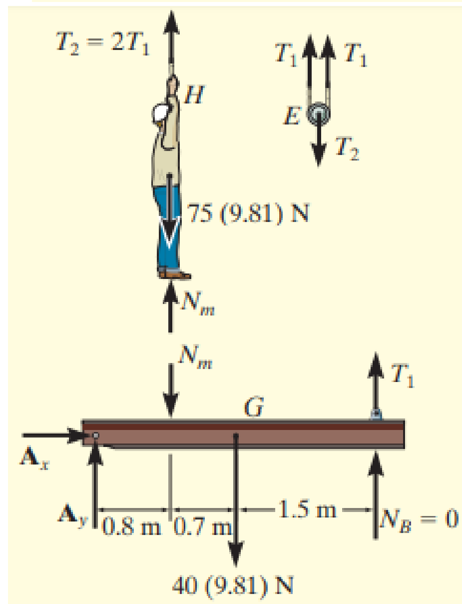
Example: The 75-kg man in figure a attempts to lift the 40-kg uniform beam off the roller support at B. Determine the tension developed in the cable attached to B and the normal reaction of the man on the beam when this is about to occur.



(a)

Solution:

Free-Body Diagrams. The tensile force in the cable will be denoted as T_1. The free-body diagrams of the pulley E, the man, and the beam are shown in figure b. Since the man must lift the beam off the roller B then NB = 0. When drawing each of these diagrams, it is very important to apply Newton's third law.



(b)

Equations of Equilibrium. Using the free-body diagram of pulley E,

$$+\uparrow \Sigma F_y = 0; \qquad 2T_1 - T_2 = 0 \quad \text{or} \quad T_2 = 2T_1 \qquad (1)$$

Referring to the free-body diagram of the man using this result,

$$+\uparrow \Sigma F_y = 0 \qquad N_m + 2T_1 - 75(9.81)\text{ N} = 0 \qquad (2)$$

Summing moments about point A on the beam,

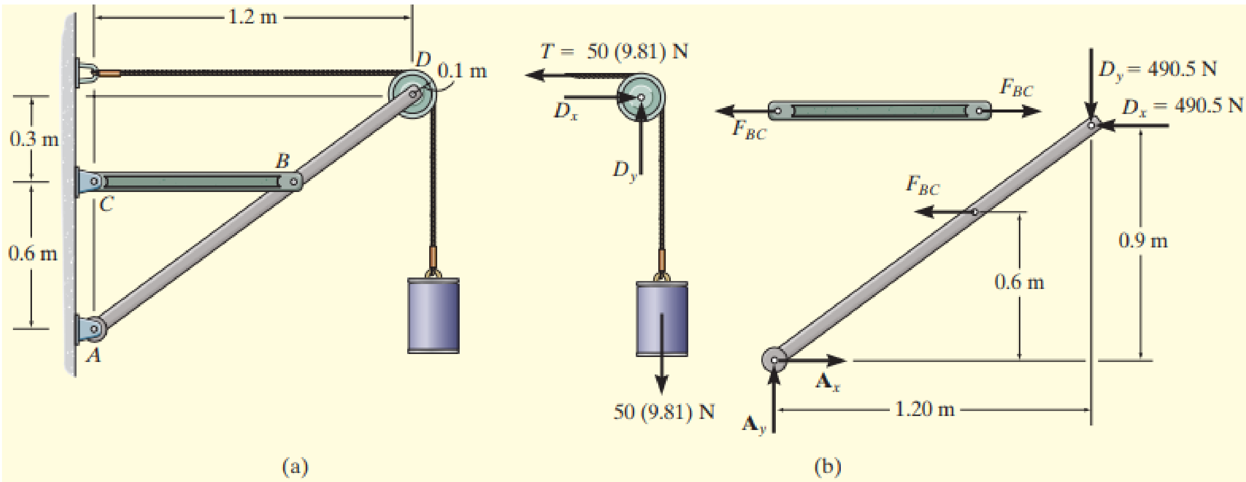
$$\circlearrowleft + \Sigma M_A = 0; \quad T_1(3\text{ m}) - N_m(0.8\text{ m}) - [40(9.81)\text{ N}]\,(1.5\text{ m}) = 0 \qquad (3)$$

Solving Eqs. 2 and 3 simultaneously for T_1 and N_m, then using Eq. (1) for T_2, we obtain

$$T_1 = 256\text{ N} \qquad N_m = 224\text{ N} \qquad T_2 = 512\text{ N}$$

Example: The frame in figure a supports the 50-kg cylinder. Determine the horizontal and vertical components of reaction at A and the force at C.

Solution:



Free-Body Diagrams. The free-body diagram of pulley D, along with the cylinder and a portion of the cord (a system), is shown in figure b. Member BC is a two-force member as indicated by its free-body diagram. The free-body diagram of member ABD is also shown.

Equations of Equilibrium. We will begin by analyzing the equilibrium of the pulley. The moment equation of equilibrium is automatically satisfied with T = 50(9.81) N, and so

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad D_x - 50(9.81)\text{ N} = 0 \quad D_x = 490.5\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad D_y - 50(9.81)\text{ N} = 0 \quad D_y = 490.5\text{ N}$$

Using these results, FBC can be determined by summing moments about point A on member ABD.

$$\circlearrowleft + \Sigma M_A = 0;\ F_{BC}(0.6\text{ m}) + 490.5\text{ N}(0.9\text{ m}) - 490.5\text{ N}(1.20\text{ m}) = 0 \quad F_{BC} = 245.25\text{ N}$$

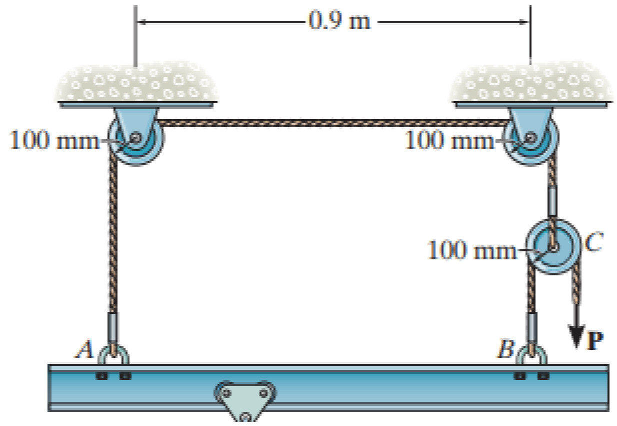
Now A_x and Ay can be determined by summing forces.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 245.25\text{ N} - 490.5\text{ N} = 0 \quad A_x = 736\text{ N}$$

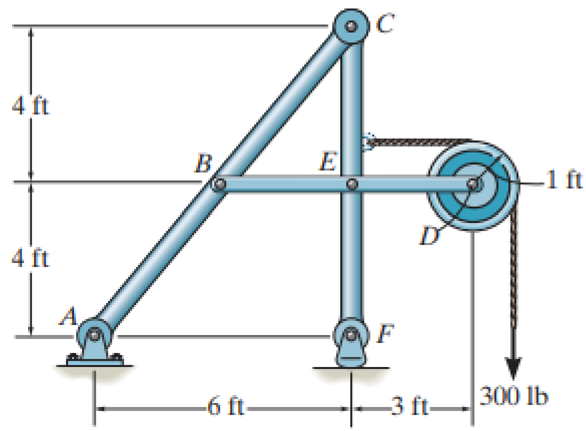
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 490.5\text{ N} = 0 \quad A_y = 490.5\text{ N}$$

H.W:

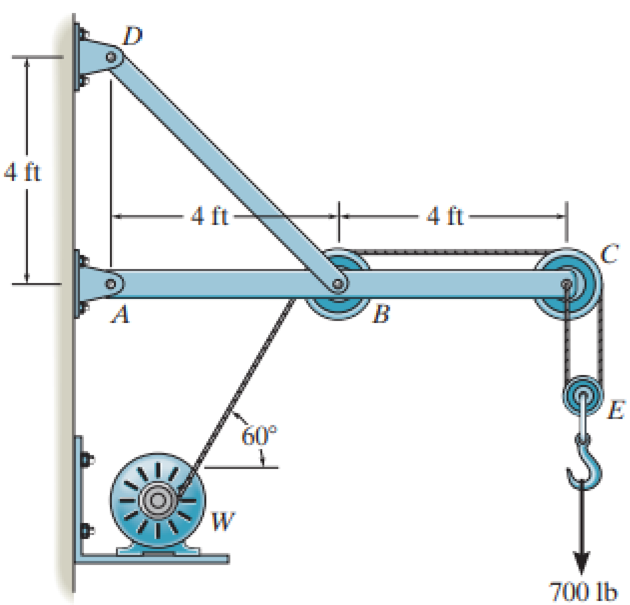
1. Determine the force P needed to lift the load. Also, determine the proper placement x of the hook for equilibrium. Neglect the weight of the beam.



2. Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF



3. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W? The jib ABC has a weight of 100 lb and member BD has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



Center of Gravity and Centroid

CHAPTER OBJECTIVES:

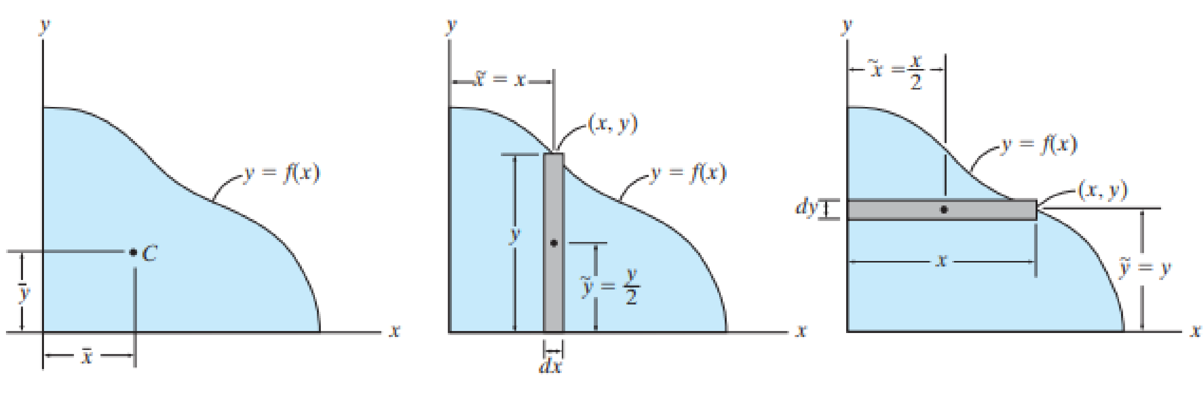
- To discuss the concept of the center of gravity, center of mass, and the centroid.
- To show how to determine the location of the center of gravity and centroid for a system of discrete particles and a body of arbitrary shape.
- To use the theorems of Pappus and Guldinus for finding the surface area and volume for a body having axial symmetry.
- To present a method for finding the resultant of a general distributed loading and show how it applies to finding the resultant force of a pressure loading caused by a fluid.

Centroid of an Area

If an area lies in the x–y plane and is bounded by the curve y = f (x), as shown in Figure a, then its centroid will be in this plane and can be determined from integrals, namely,

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

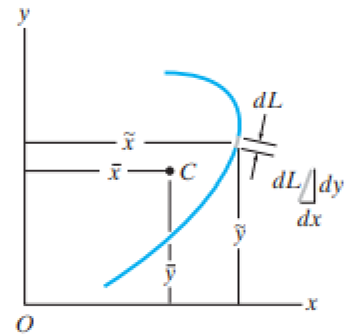
These integrals can be evaluated by performing a single integration if we use a rectangular strip for the differential area element. For example, if a vertical strip is used, Figure b, the area of the element is dA = y.dx, and its centroid is located at x = x and y =



(a) (b) (c)

y/2. If we consider a horizontal strip, figure C, then dA = x dy, and its centroid is located at x = x/2 and y = y.

Centroid of a Line: If a line segment (or rod) lies within the x-y plane and it can be described by a thin curve y = f (x), figure a, then its centroid is determined from



(a)

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL}$$

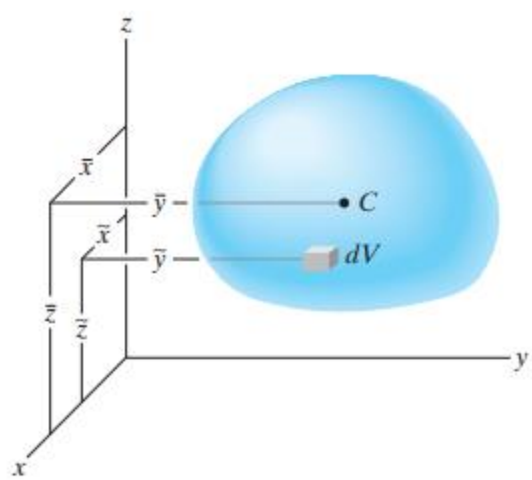
Here, the length of the differential element is given by the Pythagorean theorem, dL= $((dx)^2 + (dy)^2)^{1/2}$, which can also be written in the form

$$dL = \sqrt{\left(\frac{dx}{dx}\right)^2 dx^2 + \left(\frac{dy}{dx}\right)^2 dx^2} \qquad = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$$

Or:

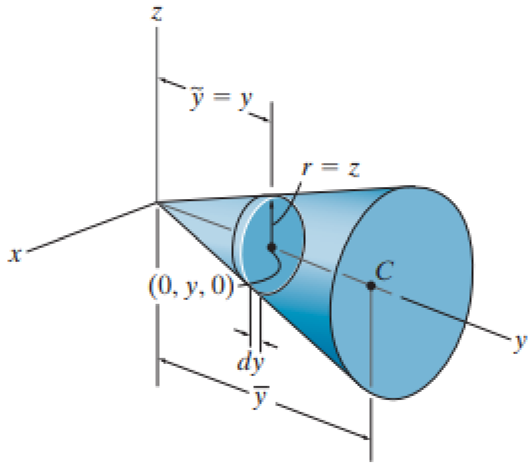
$$dL = \sqrt{\left(\frac{dx}{dy}\right)^2 dy^2 + \left(\frac{dy}{dy}\right)^2 dy^2} \qquad = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

Centroid of a Volume. If the body in Figure is made from a homogeneous material, then its density r (rho) will be constant. Therefore, a differential element of volume dV has a mass dm = r dV. Substituting this into Eqs. 9–2 and canceling out r, we obtain formulas that locate the centroid C or geometric center of the body; namely



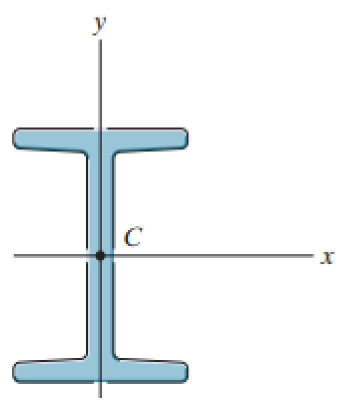
$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

These equations represent a balance of the moments of the volume of the body. Therefore, if the volume possesses two planes of symmetry, then its centroid must lie along the line of intersection of these two planes. For example, the cone in Figure has a centroid that lies on the y axis so that x = z = 0. The location y can be found using a single integration by choosing a differential element represented by a thin disk having a thickness dy and radius r = z. Its volume is $dV = \pi r^2\, dy = \pi z^2\, dy$ and its centroid is at x = 0 , y = y , z = 0 .



Important Points

- The centroid represents the geometric center of a body. This point coincides with the center of mass or the center of gravity only if the material composing the body is uniform or homogeneous.
- Formulas used to locate the center of gravity or the centroid simply represent a balance between the sum of moments of all the parts of the system and the moment of the "resultant" for the system.
- In some cases the centroid is located at a point that is not on the object, as in the case of a ring, where the centroid is at its center. Also, this point will lie on any axis of symmetry for the body,



Procedure for Analysis: The center of gravity or centroid of an object or shape can be determined by single integrations using the following procedure.

Differential Element.

• Select an appropriate coordinate system, specify the coordinate axes, and then choose a differential element for integration.

• For lines the element is represented by a differential line segment of length dL.

• For areas the element is generally a rectangle of area dA, having a finite length and differential width.

• For volumes the element can be a circular disk of volume dV, having a finite radius and differential thickness.

• Locate the element so that it touches the arbitrary point (x, y, z) on the curve that defines the boundary of the shape.

Size and Moment Arms.

• Express the length dL, area dA, or volume dV of the element in terms of the coordinates describing the curve.

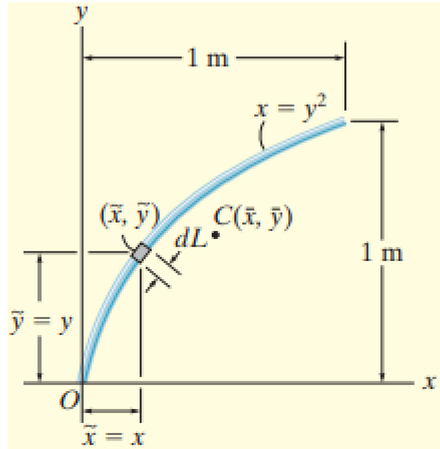
• Express the moment arms x, y, z for the centroid or center of gravity of the element in terms of the coordinates describing the curve.

Integrations.

• Substitute the formulations for x, y, z and dL, dA, or dV into the appropriate equations

• Express the function in the integrand in terms of the same variable as the differential thickness of the element.

• The limits of the integral are defined from the two extreme locations of the element's differential thickness, so that when the elements are "summed" or the integration performed, the entire region is covered.



Example: Locate the centroid of the rod bent into the shape of a parabolic arc as shown in Figure.

Solution:

Differential Element. The differential element is shown in figure. It is located on the curve at the arbitrary point (x, y).

Area and Moment Arms. The differential element of length dL can be expressed in terms of the differentials dx and dy using the Pythagorean theorem.

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$$

Since $x = y^2$, then $dx/dy = 2y$. Therefore, expressing dL in terms of y and dy, we have

$$dL = \sqrt{(2y)^2 + 1}\, dy$$

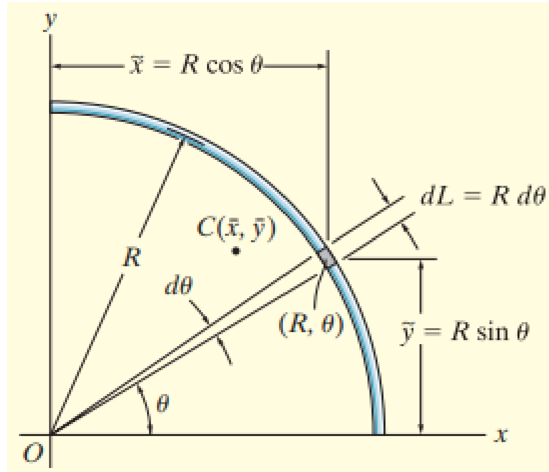
As shown, the centroid of the element is located at x = x, y = y

Integrations. Applying line centroid equation and using the integration formula to evaluate the integrals, we get:

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{\int_0^{1\text{ m}} x\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{ m}} \sqrt{4y^2 + 1}\, dy} = \frac{\int_0^{1\text{ m}} y^2\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{ m}} \sqrt{4y^2 + 1}\, dy} = \frac{0.6063}{1.479} = 0.410 \text{ m}$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^{1\text{ m}} y\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{ m}} \sqrt{4y^2 + 1}\, dy} = \frac{0.8484}{1.479} = 0.574 \text{ m}$$

Example: Locate the centroid of the circular wire segment as shown.

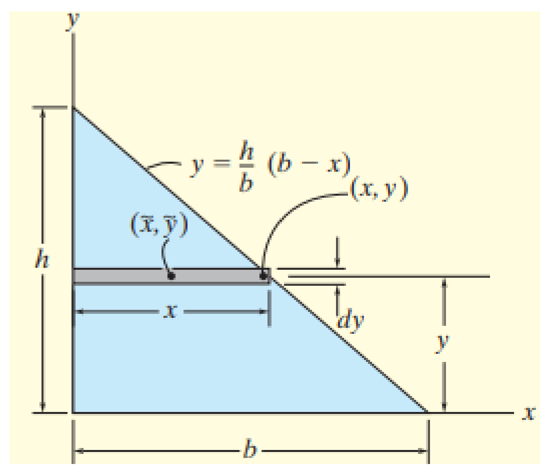


SOLUTION: Polar coordinates will be used to solve this problem since the arc is circular. **Differential Element**. A differential circular arc is selected as shown in the figure. This element lies on the curve at (R , θ). Length and Moment Arm. The length of the differential element is dL = R dθ, and its centroid is located at x = R cos θ and y = R sin θ. Integrations. Applying the equation of line centroid and integrating with respect to θ, we obtain:

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{\pi/2} (R\cos\theta)R\,d\theta}{\int_0^{\pi/2} R\,d\theta} = \frac{R^2\int_0^{\pi/2} \cos\theta\,d\theta}{R\int_0^{\pi/2} d\theta} = \frac{2R}{\pi}$$

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_0^{\pi/2} (R\sin\theta)R\,d\theta}{\int_0^{\pi/2} R\,d\theta} = \frac{R^2\int_0^{\pi/2} \sin\theta\,d\theta}{R\int_0^{\pi/2} d\theta} = \frac{2R}{\pi}$$

Example: Determine the distance y measured from the x axis to the centroid of the area of the triangle shown.



SOLUTION:

Differential Element. Consider a rectangular element having a thickness dy, and located in an arbitrary position so that it intersects the boundary at (x, y), Fig. 9–10 .

Area and Moment Arms. The area of the element is dA = x dy = (b/ h) (h - y) dy, and its centroid is located a distance y = y from the x axis.

Integration. Applying the area equation of centroid and integrating with respect to y yields

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^h y\left[\frac{b}{h}(h-y)\, dy\right]}{\int_0^h \frac{b}{h}(h-y)\, dy} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} \quad = \frac{h}{3}$$

Example: Locate the centroid for the area of a quarter circle shown in Fig

SOLUTION

Differential Element. Polar coordinates will be used, since the boundary is circular. We choose the element in the shape of a triangle. (Actually the shape is a circular sector; however, neglecting higher-order differentials, the element becomes triangular.) The element intersects the curve at point (R,θ).

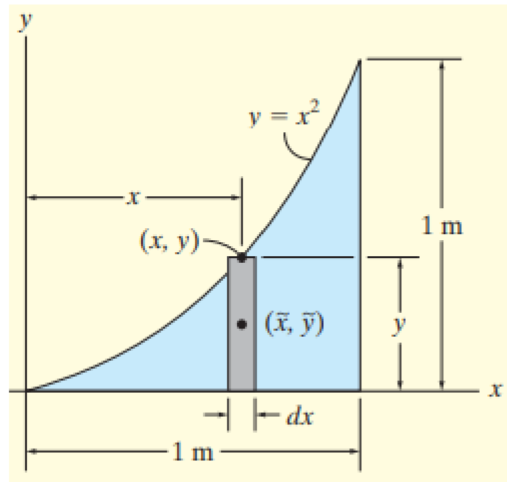
Area and Moment Arms. The area of the element is dA = 0.5(R) (R dθ) = 0.5R^2 dθ and using the results of previous example, the centroid of the (triangular) element is located at x = (2/3) R cos θ, y = (2/3) R sin θ.

Integrations. Applying area equation and integrating with respect to θ, we obtain:

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^{\pi/2}\left(\frac{2}{3}R\cos\theta\right)\frac{R^2}{2}d\theta}{\int_0^{\pi/2}\frac{R^2}{2}d\theta} = \frac{\left(\frac{2}{3}R\right)\int_0^{\pi/2}\cos\theta\, d\theta}{\int_0^{\pi/2} d\theta} = \frac{4R}{3\pi}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^{\pi/2}\left(\frac{2}{3}R\sin\theta\right)\frac{R^2}{2}d\theta}{\int_0^{\pi/2}\frac{R^2}{2}d\theta} = \frac{\left(\frac{2}{3}R\right)\int_0^{\pi/2}\sin\theta\, d\theta}{\int_0^{\pi/2} d\theta} = \frac{4R}{3\pi}$$

Example: Locate the centroid of the area shown in figure a.



(a)

Solution:

Differential Element. A differential element of thickness dx is shown in Figure a. The element intersects the curve at the arbitrary point (x, y), and so it has a height y.

Area and Moment Arms. The area of the element is dA = y dx, and its centroid is located at x = x, y = y/2.

Integrations. Applying area equation for centroid and integrating with respect to x yields:

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} xy\, dx}{\int_0^{1\text{ m}} y\, dx} = \frac{\int_0^{1\text{ m}} x^3\, dx}{\int_0^{1\text{ m}} x^2\, dx} = \frac{0.250}{0.333} = 0.75\text{ m}$$

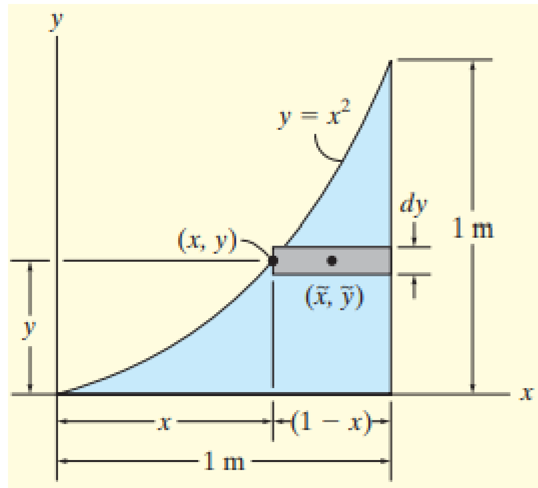
$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} (y/2)y\, dx}{\int_0^{1\text{ m}} y\, dx} = \frac{\int_0^{1\text{ m}} (x^2/2)x^2\, dx}{\int_0^{1\text{ m}} x^2\, dx} = \frac{0.100}{0.333} = 0.3\text{ m}$$

SOLUTION II:

Differential Element. The differential element of thickness dy is shown in Figure b . The element intersects the curve at the arbitrary point (x, y), and so it has a length (1 - x). **Area and Moment Arms**. The area of the element is dA = (1 - x) dy, and its centroid is located at

$$\tilde{x} = x + \left(\frac{1 - x}{2}\right) = \frac{1 + x}{2},\ \tilde{y} = y$$

Integrations. Applying area equation and integrating with respect to y , we obtain

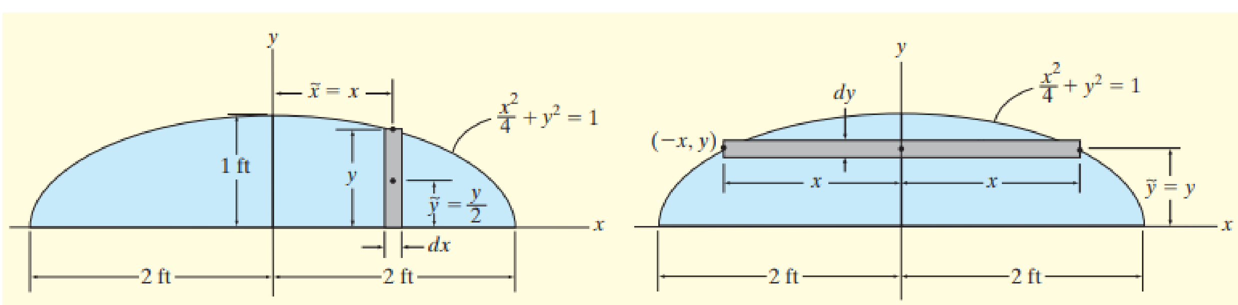


(b)

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} [(1+x)/2](1-x)\, dy}{\int_0^{1\text{ m}} (1-x)\, dy} = \frac{\frac{1}{2}\int_0^{1\text{ m}} (1-y)\, dy}{\int_0^{1\text{ m}} (1-\sqrt{y})\, dy} = \frac{0.250}{0.333} = 0.75\text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} y(1-x)\, dy}{\int_0^{1\text{ m}} (1-x)\, dy} = \frac{\int_0^{1\text{ m}} (y - y^{3/2})\, dy}{\int_0^{1\text{ m}} (1-\sqrt{y})\, dy} = \frac{0.100}{0.333} = 0.3\text{ m}$$

Example: Locate the centroid of the semi-elliptical area shown in figure a



(a) (b)

Solution:

Differential Element. The rectangular differential element parallel to the y axis shown shaded in Fig. 9–13 a will be considered. This element has a thickness of dx and a height of y.

Area and Moment Arms. Thus, the area is dA = y dx, and its centroid is located at x = x and y = y/2.

Integration. Since the area is symmetrical about the y axis, x = 0

Applying area equation with $y = (1 - x^2/4)^{0.5}$, we have:

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_{-2\text{ ft}}^{2\text{ ft}} \frac{y}{2}(y\,dx)}{\int_{-2\text{ ft}}^{2\text{ ft}} y\,dx} = \frac{\frac{1}{2}\int_{-2\text{ ft}}^{2\text{ ft}}\left(1 - \frac{x^2}{4}\right)dx}{\int_{-2\text{ ft}}^{2\text{ ft}}\sqrt{1 - \frac{x^2}{4}}\,dx} = \frac{4/3}{\pi} = 0.424\text{ ft}$$

SOLUTION II

Differential Element. The shaded rectangular differential element of thickness dy and width 2 x, parallel to the x axis, will be considered, figure b.

Area and Moment Arms. The area is dA = 2x dy, and its centroid is at x = 0 and y = y.
Integration. Applying the equation of area, with $x = 2(1 - y^2)^{0.5}$, we have:

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^{1\text{ ft}} y(2x\,dy)}{\int_0^{1\text{ ft}} 2x\,dy} = \frac{\int_0^{1\text{ ft}} 4y\sqrt{1 - y^2}\,dy}{\int_0^{1\text{ ft}} 4\sqrt{1 - y^2}\,dy} = \frac{4/3}{\pi}\text{ ft} = 0.424\text{ ft}$$

H.W: Locate the centroid of the following shapes: